Brand Reallocation and Market Concentration

Jeremy Pearce[*] Liangjie Wu[†] [‡§]

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Abstract

This paper studies the interaction of customer capital and firm productivity through brand reallocation across firms. We link USPTO trademarks with retail prices and quantities of brands to show that market concentration at the firm level is primarily driven by the growth of firms' existing brands and the acquisition of brands from competitors. These acquired or reallocated brands experience increases in both revenues and prices. To study the interaction between productivity and customer capital, we develop a firm dynamics model with the brand as transferable customer capital, heterogeneous firm productivity, and variable markups. A novel insight of the model is the role of brand capital in generating a dynamic mismatch between productivity and market share, as unproductive incumbents can maintain large market share through brand reallocation and turn the rank correlation between productivity and market share negative and lead to first-order welfare losses. Quantitatively, we find an aggregate efficiency gain from brand reallocation but distortions due to the mismatch between brand capital and productivity; many markets exhibit persistent inefficient leadership. As opposed to size-based subsidies, entry subsidies can reduce inefficient reallocation and promote efficient reallocation, sorting brand capital to productive young firms instead of inefficient incumbents.

Key Words: Firm Dynamics, Productivity, Market Concentration, Product Innovation, Reallocation, Mergers & Acquisitions, Brands, Trademarks, Intangible Assets

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[*]Federal Reserve Bank of New York, jeremy.pearce@ny.frb.org

[†]Einaudi Institute for Economics and Finance, liangjie.wu@eief.it;

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1 Introduction

Brands are an essential intangible asset for firms. The top 100 brands in the US economy were worth over $4 Trillion in 2021, and the relative value of brands to traditional capital has been growing over time (Bronnenberg et al., 2022).[1] We define a brand as a tradable form of customer capital; it is the means by which a firm accesses customers through their recognition of the symbol corresponding to underlying products. The accumulation of customer capital takes time. Besides building customer capital gradually (Gourio and Rudanko, 2014), firms can also directly acquire customer capital through brand acquisition or *brand reallocation*. When brand ownership is transferred across firms through reallocation, the customer capital or brand capital transferred is now matched with the productivity and strategy of the acquiring firm. Brand reallocation can thus not only provide insights into the nature of brand capital but also broader questions in firm dynamics, concentration, and productivity. What is the dynamic role of brands in firm competition and what are the macroeconomic implications of brand reallocation on aggregate productivity and efficiency?

We start with three facts to motivate the discussion and provide a benchmark for answering the paper's main questions.

- **Fact 1: Persistent leadership.** Firms persistently lead their market groups and have outsized market share relative to the median firm (e.g., 3000-times market share).
- **Fact 2: Reallocation matters.** Brand reallocation represents a significant amount of across-firm market share reallocation (more than 25%) and goes to larger firms.
- **Fact 3: Event studies.** When brands are reallocated, sales and prices both increase.

To unify these facts with our research question, we proceed in three steps. We start by developing a firm dynamics model where firms have nontransferable productivity and transferable brand capital. Firms' pursuit of profit in reallocating brand ownership delivers a classic efficiency-markup tradeoff that interacts with a novel dynamic mismatch between brand capital and productivity. A firm with high brand capital and low productivity can maintain a higher market share than its more productive competitors through advertising and brand reallocation. Second, we construct a novel dataset to empirically analyze firms' brands in their trademark holdings and the corresponding sales and prices of the underlying products at the retail level. This novel brand-firm dataset enables our study of market share as jointly determined by fixed firm productivity and transferable brands. We use the data to present new facts on the interaction of the evolution of market share, the role of brand capital, and brand reallocation events. Third, we quantify the model, study the brand reallocation events, and perform policy counterfactuals. Theoretically, brand reallocation can be inefficient or efficient depending on the market specifics. In our estimation, we find that brand reallocation is efficient relative to a world without it. However, the effects are heterogeneous across markets: some markets exhibit evidence of persistent market share leaders that are inefficient. On the whole, shutting down brand reallocation decreases welfare by 10%; to induce efficient reallocation from unproductive to productive firms, entry subsidies are an effective policy.

[1] The estimated $4.14 Trillion represents 0.47 of the size of the total value of Property, Plant, and Equipment at the same firms.

The theoretical model opens with a simple framework of a market with two competing firms that differ in their exogenous productivity and customer base or brand capital, which they can exchange or reallocate. Productivity evolves exogenously, with entering firms replacing technological followers, while brand capital evolves both exogenously, through productivity catch-up, and endogenously, through advertising and brand reallocation. When firms transact brands, they can jointly increase profits from sorting brand capital to the better firm but also, in oligopolistic competition with variable markups, through consolidating monopoly power. Thus, two incentives arise in brand reallocation: (1) a *productive* motive from sorting brand capital to the more productive firm, related to findings in M&A (David, 2020) and (2) a *strategic* motive as the larger firm is able to appropriate more profit from each unit of revenue (modeled following Atkeson and Burstein, 2008). Firms are only concerned with their own profits, which creates externalities in the marketplace: reducing active competitors increases profits and reduces the ability of customers to substitute away from the largest firm. In addition to the static distortion due to markups (Edmond et al., 2015), the externalities can also create a dynamic mismatch between firm productivity and brand capital. An unproductive firm could have a large market share and a high growth rate for an extended period due to its accumulated brand capital from the past, which persists through the acquisition of new brands. The insight on dynamic mismatch of brand capital and productivity has important empirical and policy implications.

The model leads to two empirical questions. Are brands and brand reallocation an important driver of market share dispersion? Is there evidence of productive and strategic motives in reallocation? To answer these questions, we construct a novel dataset of firms' brands in their trademark holdings, which we link to retail prices and sales. Trademarks are the intellectual property protecting brand capital for the owning firm by not allowing competitors to capture brand capital through imitation. In line with our theory, market access occurs through customer recognition. The US Patent and Trademark Office (USPTO) defines a trademark as "any word, phrase, symbol, design, or a combination of these things that identifies your goods or services. It's *how customers recognize you in the marketplace* and distinguish you from your competitors." The USPTO records all federal trademarks that are registered, canceled, or reassigned. Through this dataset, we are able to locate brand reallocation events via trademark reassignment across firms. On average, 2% of trademarks are reallocated across firms per year. This rate is similar across all sectors, indicating that brand reallocation is a pervasive feature of the economy.[2]

We use the linked trademark-brand dataset to study the joint evolution of firms and brands. We first decompose the variance of market share at the firm level, and show how the variance in sales dispersion at large firms is most driven by reallocation. For firms with more than 1% market share in a given product group, 50% of the firms' variance in sales is due to reallocation. The role of reallocation is especially stark for the largest firms; we show that without reallocation the largest firms would be 10% smaller after a decade. Relative to similar non-reallocated brands, we find that the average reallocated brand experiences an increase in revenue by 40% and an increase in price by 4% in the three years post reallocation. Interpreted through the lens of our theory, this suggests both a productive motive (sales $\uparrow$) and a strategic motive (price $\uparrow$). Leveraging variation at the retailer level, we find the price effect is

[2]There are 46 NICE codes that indicate the domain of the trademark. These NICE codes can be mapped to NAICS codes. See Kost et al. (2019) for more detailed analysis on brand reallocation across NAICS categories.

stronger for retail chains where the buying firm has a larger market share, consistent with the assumption that a firm's markup increases in its market share, which has been found in previous literature (e.g., Amiti et al., 2019).

To quantify the welfare incidence of brand reallocation, we extend our simple model to a general equilibrium setting with richer features. In the full model, we introduce entrants who compete against the existing duopolists, an endogenous labor supply, advertising at the firm-level, and a distribution of productivity gaps. We stress here the importance of both entry, which provides a competitive force that shapes the market, and advertising, which firms can invest in as another form of building customer base. We show there is a unique steady-state equilibrium that depends on the distribution of brand capital, productivity, and fundamental parameters. We quantify this model to ask two questions regarding brand reallocation: how big is the mismatch between brand capital and productivity, and how does understanding the joint determination customer base and productivity change the current policy remedies to market power?

We direct our attention to the distribution of mismatch across markets. In the stationary distribution, we find that X% of markets are those in which the brand leader is not the productivity leader. Not all of these markets experience inefficient reallocation, which would perpetuate the inefficiency. Y% of markets are persistently inefficient, as firms with high brand capital but low productivity maintain market share through advertising and brand acquisitions. The persistence and degree of the mismatch depends specific features of markets, such as speed of brand maturity. In markets where brands mature quickly, brand capital has more capability of catching up with productivity, leading to alignment of productivity leader and market leader. In markets where brands mature slowly, unproductive incumbents have a greater chance of maintaining brand capital through past success and brand acquisitions. These patterns have important implications for overall market efficiency in a broad set of markets beyond the ones discussed in this paper.

We last turn our framework to the firm dynamics literature and policy discussions. Most results in the literature (such as Edmond et al., 2023 and Boar and Midrigan, 2019) focus on markets where only productivity determines market share and find that subsidizing large firms restores efficiency in markets with monopoly. We study a variety of standard and new policies in our environment where productivity and customer base jointly determine market share. A subsidy to large firms can exacerbate inefficiencies in our environment, as large firms may simply be large due to customer accumulation and may be less efficient than their small competitors. Given the significant share of markets where the productivity and market share are negatively correlated, this policy is indeed inferior to many other policies to ameliorate the inefficiency. Subsidizing entry, as one example, can alleviate this distortion by promoting market access and, through reducing the market share of the duopolists, promote efficient brand reallocation. A last key point is the role of heterogeneity in markets. Because markets are shaped by different forces depending on fundamentals and history, a good policy is one market may look very different in another market. We discuss these details in Section 4 and the conclusion.

Related Literature. This paper builds on and contributes to several literatures: the macroeconomics of M&A and technology transfers; the study of firm dynamics, product dynamics and productivity; the study of concentration, innovation, and firm profitability; and the study of brands and branding.

The macroeconomic implications of mergers and acquisitions (M&A) and technology acquisitions have received rising interest. David (2020) studies the aggregate implications of M&A and finds M&A increases overall efficiency in sorting productive assets to productive firms, an important channel for efficiency gains in our framework. Many other papers study the implications of technology and patent transfer in particular (Eaton and Kortum, 1996, Akcigit et al., 2016, Shi and Hopenhayn, 2017). One reason for firm acquisition is the acquisition of customer capital on the demand side, an important ingredient in firm value and market share (Dinlersoz and Yorukoglu, 2012; Gourio and Rudanko, 2014). Whether this customer capital is sorted to efficient firms relates to studies on strategic versus efficient transactions in the market for firms and IP assets (Spearot, 2012, Abrams et al., 2019, Cunningham et al., 2021). Recent papers have focused on this in a dynamic setting (Cavenaile et al., 2021 and Fons-Rosen et al., 2021). We believe we are the first paper to place tradable market access, in the form of brands, at the center of this discussion.

Hottman et al. (2016) study multi-product firms and find that the "appeal" of products explains a large share of sales variation across firms. Our paper points to how a large share of appeal is located in the *brand*, how much of this is tradable, and plays an important dynamic role. Argente et al. (2018, 2020b) and Jaravel (2018) explore how product creation and destruction are pervasive in product markets. Smith and Ocampo (2022) document the rise of market concentration, a significant force in product markets. Further, Argente et al. (2021) and Einav et al. (2021) document that the expansion of product sales is primarily due to expansion of the customer base. We connect these important empirical insights to a firm's decision when it holds many products in the form of brands. This connects to a broader literature in productivity and welfare that asks about the role of products (Bils and Klenow, 2001; Broda and Weinstein, 2006) the role of firm heterogeneity (Syverson, 2004a; Hsieh and Klenow, 2009), misallocation (Restuccia and Rogerson, 2017; David and Venkateswaran, 2019) and market power (Syverson, 2004b; Melitz and Ottaviano, 2008). Our goal in this paper is to develop insights on how productivity interfaces with demand-side factors which economists have begun to put at center stage in the study of productivity (Foster et al., 2008; Syverson, 2011, Sterk et al., 2021, and Michelacci et al., 2022). Productivity, or the efficiency of the transformation of inputs into outputs, is a supply-side concept. In standard models of firms, large firms have better expertise or technology and produce more due to this supply-side advantage. However, there is growing evidence that demand or appeal plays a large role in firm size distribution (Hottman et al., 2016). If a firm's higher market share comes from the consolidation of customer capital, it may generate mismeasurements in productivity. In this paper, we analyze a case where the correlation between revenue productivity and quantity productivity could take the *opposite sign* due to demand-side consolidation. We find that this is quantitatively a significant share of product markets.

Brands and brand reallocation also provide another avenue to study the link between firm dynamics, market concentration, and markups (such as De Loecker and Eeckhout, 2018; De Loecker et al., 2020; Gutiérrez and Philippon, 2017; Eggertsson et al., 2018; Hall, 2018; Autor et al., 2020). Kehrig and Vincent (2018) find evidence of rising concentration and reduction in the labor share and connect this to previous marketing expenditures. Boar and Midrigan (2019) link markups to inequality. Bornstein and Peter (2022) link markups to misallocation at the customer level. We connect these discussions to firm dynamics and competition by building on the long literature of creative destruction (Aghion and Howitt, 1992, Aghion et al., 2001, Peters, 2020, and Liu et al., 2022), augmenting this with literature on entry and firm development (Jovanovic, 1982; Hopenhayn, 1992). In our case, the competition over market share exhibits business stealing effects that firms ideally want to avoid. Some papers in this tradition focus on the links between factor or labor reallocation and growth (Acemoglu et al., 2018; Garcia-Macia et al., 2019), while we focus on tradable intangible capital. Jones and Williams (1998, 2000) study how markups and innovation interact to determine over- or under-investment in the creation of new products. Baslandze et al. (2023) study the introduction of new products empirically and theoretically. Edmond et al. (2015) focus on the markup channel, as large firms can leverage their large market share to charge high markups, a feature we explore in this paper. Amiti et al. (2019) find that strategic considerations occur in large firms' pricing decisions but not small firms' pricing decisions. To model this mechanism, our paper builds on Atkeson and Burstein (2008), who introduce an oligopolistic competition model with large multi-product firms where concentration and markups are jointly determined. In speaking to the role of tradable brand capital, we address the interaction of market concentration with intangible assets and customer acquisition (Bhandari and McGrattan, 2020) which shows up in advertising investments (Cavenaile and Roldan-Blanco, 2021; Greenwood et al., 2021) and can affect aggregate growth (Ignaszak and Sedlácek, 2022; Cavenaile et al., 2023). This connects more broadly to a set of papers have directed attention to the role of intangible forces in shaping modern markets, concentration, and growth (Akcigit and Ates, 2019, 2021, Haskel and Westlake, 2017, Crouzet and Eberly, 2019, Syverson, 2019, De Ridder, 2024).

Lastly, we bring insights from the literature on brands and branding to the macroeconomic debates on concentration, markups, and productivity. Brands have long been known to be an important source of firm values (e.g., Braithwaite, 1928 on brands, and Brown, 1953 on trademarks). Bain (1956) noted that "(t)he advantage to established sellers accruing from buyer preferences for their products as opposed to potential-entrant products is on the average larger and more frequent in occurrence at large values than any other barrier to entry." Nelson (1970) pointed out that market power is closely linked to customer attention. Theoretically, brands can generate persistent profits in markets with imperfect information (Schmalensee, 1978, Shapiro, 1983 and Schmalensee, 1982). The power of branding has been detailed empirically as consumer brand preferences are quite persistent (e.g., in Bronnenberg et al., 2009, 2012) and thus provide firms significant and tradable value (Tadelis, 1999). Trademarks serve as the central institution that links the property right to the brand (Economides, 1988). Recently, more work has developed insights on firm dynamics with trademarks (Dinlersoz et al., 2018, Heath and Mace, 2019, Castaldi, 2019 and Kost et al., 2019 stress the high degree of activity in the market for trademarks). This current paper builds on these ideas by linking brand capital to the market shares of firms and studying how brands can be reallocated across firms. This adds tradability to classic work in industrial organization on advertising and market structure (Butters, 1977, Grossman and Shapiro, 1984, Sutton, 1991, and Stegeman, 1991).

2 A Theory of Brand Development and Reallocation

To frame our investigation into the USPTO Trademark and Nielsen scanner data, we introduce a theory of brands and brand reallocation that can be brought to some core features in the data. We start with an equilibrium model of competition among oligopolistic firms with heterogeneous nontransferable productivity and transferable brand capital. We use this environment to explore a simple analytical case that highlights some core predictions of our model that can be brought to the data to study the interaction between brand development and reallocation, productivity, and markups. In Section 4, we bring the predictions to the empirical evidence and quantify the general model.

2.1 Environment

We discuss the environment which has a representative household, a set of product groups, two duopolists within each product group, and a set of fringe firms. We focus on the role of productivity across firms, brand capital, competition, and aggregation.

Household, Firms, and Brands Time is continuous. There is a representative household that supplies L_t units of labor and consumes branded products in a measure 1 of product groups. The product groups are indexed by $k \in [0,1]$. The real consumption of the household, C_t, is aggregated across product groups by a Cobb-Douglas aggregator:

$$C_t = \exp\left(\int_0^1 \log c_{kt} dk\right), \tag{1}$$

$$c_{kt} = \left(\sum_{j=1,2} e^{\frac{1}{\sigma}(a^D_{jt}+b_{jt})} d_{jkt}^{\frac{\sigma-1}{\sigma}} + n\bar{d}_{kt}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \ \sigma > 1. \tag{2}$$

In equation (1), c_{kt} is the consumption from product group k. Within each product group, there are two large firms (which we refer to as duopolists) and a measure n of fringe producers, where n is endogenously determined. The consumption at the group level is an constant-elasticity-of-substitution (CES) aggregator from the consumption on the products from firm j, d_{jkt}, and the consumption on fringe products, $\bar{d}_{kt}$.

The model takes a similar structure to innovation models with duopolistic competition, such as Akcigit and Ates (2021) and Liu et al. (2022), but departs through our assumptions on the determinants of firm heterogeneity. The classic models of firm innovation focus on one dimension of heterogeneity, firm productivity. We also assume firms differ in their productivity, but also they differ in their brand capital stock. These two components of heterogeneity depict the fact that firms have two important and distinct inputs into their value: technology and customer awareness. The following paragraphs define these determinants and their evolution in steps.

Productivity. When it comes to the technology side, we focus on how firms differ in their appeal and productivity. The firm appeal a^D_{jt} captures the difference of firms in their images to customers and distributional efficiency, as modeled in, for example, Hottman et al. (2016) and Argente et al. (2020c). Firms also differ in how efficient they are in utilizing labor in production, a^L_{jt}. We refer to the summation of these two terms as *firm productivity* : $a_{jt} \equiv a^D_{jt} + a^L_{jt}$. The differentiation between demand-side and supply-side heterogeneity does not play a role in the theoretical results, but maps into different empirical implications on price or reveneu changes. We assume that at each instant, their is a productivity leader among the duopolists, who has productivity α_{jt}, while the other duopolists (refered to as the productivity follower) and the fringe producers all have productivity of 1. The productivity leadership changes over time, through an exogenous creative destruction process. With a Poisson arrival rate of γ, a new leader enters and improve upon the old leader with a step size α. In this event, the old productivity leader becomes the new follower and the old follower exit the market. This process is similar to endogenous growth models such as Akcigit and Ates (2019) and Peters (2020).

Brand Capital. Firms differ in their brand capital b_{jt}. Building on the theory of limited customer attention, we normalize the total brand capital to be $1 = \sum e^{\frac{1}{\sigma} b_{jt}}$. This normalization takes a zero-sum view of branding. The change of brand capital at the firm level does not directly create utility, but a reallocation between the duopolists. To see this, notice when we set the productivity gap to be one, the allocation of brand capitals do not affect consumption.

The core addition of our theory is the dynamics in brand capitals. The brand capitals change through two meachanisms. First, the productivity leader gains brand capital with a growth rate χ, through either advertising or through word-of-mouth. We refer to χ as the advertising intensity. For the baseline model, we take this intensity as given and analyze a case when it is endogenously determined as extension; Endogenously, firms also have chance to consolidate their brand capital. This *reallocation* happens with rate λ. During a brand reallocation, the duopolists can decide the new gap between their brand capitals, by paying a labor cost $R(|b' - b|)$, where R is increasing and convex with condition $R'(0) = 0$ and $R'(\infty) = \infty$.

Entry. Fringe firms have free entry, with a one-time cost of κ in labor. We assume the entry in undirected: a entering fringe firm has the same probability of entering any product group. This is a common assumption in both the creative destruction literature Klette and Kortum (2004) and in the general equilibrium models with oligopolistic competition Edmond et al. (2023).

Aggregation. All activities in the economy uses labor as the input. The labor resource constraint of the economy requires:

$$L_t = \int_0^1 \left(\sum_{j=1,2} \frac{c_{jkt}}{e^{\frac{1}{\sigma-1} a^L_{jkt}}} \right) dk + \lambda \int_0^1 R(|b'_{jkt} - b_{jkt}|) dk. + \kappa n. \tag{3}$$

The rest of the household's problem is standard. We assume the household has a flow utility of $\log C_t - L_t$. The assumption of linear labor disutility simplifies the characterization of equilibrium by assuming away the crowding out of reallocation activity to production activity. This assumption is not essential in our quantitative analysis, where it will be relaxed. The household can borrow and save in a representative portfolio of all firms, such that the aggregate profit Π_t is rebated to the household as a dividend. Define r_t as the interest rate and normalize the wage to be 1. We write the household's problem is

$$\max_{c_{jkt}, L_t} \int_0^\infty e^{-\rho t} \left(\log C_t - L_t \right) dt,$$

s.t.

$$\dot{a}_t = r_t a_t + \mathbf{L}_t + \int_0^1 \left(\sum_{j=1,2} p_{jkt} c_{jkt} + n\bar{d}\bar{p}_{kt} \right) dk + \Pi_t.$$

Oligopolistic Competition. The structure of competition has important incentives for determining the brand reallocation across firms. We assume that the duopolists take as given the aggregate price index across all product groups, $\mathbb{P}$, yet are large enough to internalize their impact on their own product group (as in Atkeson and Burstein, 2008). As a result, they charge a variable markup in the equilibrium, which is a function of their market shares, $\mu(s_j) = \frac{\epsilon(s_j)}{\epsilon(s_j)-1}$. Under Bertrand competition, the perceived elasticity is $\epsilon(s) = \sigma(1-s) + s$, while under Cournot competition, the perceived elasticity is $\epsilon(s) = \left(\frac{1}{\sigma}(1-s) + s\right)^{-1}$.

2.2 Characterization

We now turn to characterizing the equilibrium, moving from the household to the firms' pricing decisions, to the dynamic reallocation decisions. We then define equilibrium and discuss welfare.

Household Decision and Pricing Equilibrium. The optimal choice of consumption from the household leads to a standard constant elasticity of substitution (CES) demand curve for products held by firm j:

$$c_{jkt}(p) = \exp(a^D_{jkt} + b_{jkt}) \left(\frac{p}{P_{jkt}} \right)^{-\sigma} \frac{\mathbf{PC}}{P_{jkt}}, \tag{4}$$

and consumption-leisure tradeoff implies $\mathbf{PC} = 1$. One important point to note is that household demand for firm j's products is a function of the relative price of goods (p), the elasticity of substitution (σ) and the composite appeal term for the firm, which has both nontransferable (a) and transferable market access (b). This object will be important for studying the nature of reallocation and its impact on sales and prices.

The equilibrium market share of the two firms depend only on their relative productivity and relative brand gap, which we define as $z = a + b$. The z measure delivers a composite term that contains productivity, appeal, and transferable brand capital. The market share of the first firm is thus:

$$\frac{s}{1-s} = e^z \frac{\mu(s)^{1-\sigma}}{\mu(1-s)^{1-\sigma}}. \tag{5}$$

A core object of interest of our model is the joint profit of the firms, which is the market share of each firm multiplied by their profit margins, $\omega(z) = s(z)\frac{1}{\epsilon(s(z))} + (1-s(z))\frac{1}{\epsilon(1-s(z))}$. A special case is useful to highlight the static equilibrium outcomes. Suppose the competition is through Cournot and there is no entry of fringe firms $n = 0$. we are able to solve this joint profit function in closed form:

$$\omega(z) = \frac{1}{\sigma} + \left(1 - \frac{1}{\sigma}\right) \frac{1 + 2e^{z/\sigma}}{(1 + e^{z/\sigma})^2}.$$

Now we have the static return to exchange which informs the dynamic decision over the reallocation of brands.

Dynamic Decisions. In our baseline model, the only dynamic decision is the reallocation of brand capital. We denote the discounted value of the productivity leader as $V(b)$ and the discounted value of the producitvity follower as $v(b)$. For convience of characterizing the reallocation decisions, we denote their joint surplus as $\Omega(b) = V(b) + v(b)$. The leader's discount value is the solution to the following Hamiltonian-Jacobian-Bellman equation (HJB):

$$\rho V(b) = \Pi(a+b) + \lambda\beta\Omega(b) + \gamma(v(b_0) - V(b)) + \chi V'(b). \tag{6}$$

The leader receives profit $\Pi(a+b)$ every instant. With rate λ, the duopolists have chance to consolidate, where $\Omega(b)$ is the gains from trade, among which the leader receives a share β. Creative desruction occurs at rate γ, and a new duopolist enters and becomes the frontier firm, with the old frontier firm becoming the vintage firm. At rate χ, the leader's brand capital grows. The discounted value for the follower follows a similar equation, except that the follower receives zero (from existing) when the creative destruction happens:

$$\rho v(b) = \pi(a+b) + \lambda(1-\beta)\Omega(b) + \gamma(0 - v(b)) + \chi v'(b). \tag{7}$$

The joint surplus is the maximal gains from reallocation, which solves the following problem:

$$\Omega(b) = \max_{b'} V(b') + v(b') - V(b) - v(b) - R(|b' - b|).$$

Equilibrium Definition. In equation (6) The distribution of product groups with brand capital b evolves according to:

$$\dot{g}(b) = -(\lambda+\gamma)g(b) - \chi g'(b) + \lambda \int_0^1 \mathbb{I}\{B(b') = b\} g(b') db', \tag{8}$$

with a restriction $\int_0^1 g(b)db = 1$. With this distribution in hand, we now formally define a steady-state equilibrium. This definition can be extended to incorporate transitional paths, which we leave to the Appendix.

Definition 1 (Steady-state Equilibrium) *A steady-state equilibrium is a collection of static pricing equilibrium $\{s(b), n\}$, dynamic decision $\{V(b), v(b), B(b)\}$, distribution $g(b)$, aggregate consumption and labor $\{C, L\}$, and aggretate price index P such that:*

1. *(pricing equilibrium) $\{s(b), n(b)\}$ solve the static pricing equilibrium ;*
2. *(dynamic optimal) $\{V(b), v(b), B(b)\}$ solve equation (6);*
3. *(stationarity) $g(b)$ solves equation (8);*
4. *(labor supply) L*
5. *(market clear) labor market clears according to equation (3)*

The steady-state equilibrium can be computed recursively. First, the discussion so far establishes that the only aggregate variable that requires solution in iteration is n. We look for such an n that is consistent with the free-entry condition. The rest of the equilibrium objects can be computed from the perspective equations.

Welfare. The household's utility depends on both the dispersion of markups and on whether brand capital is allocated towards the productive firm. To see this, we write out the consumption from a product group, when the productivity gap is a while the brand gap is b. The discounted utility of the representitive household is as follows:

$$\mathbf{W} = -\frac{1}{\rho}\left\{ \int_0^1 \left[\log P(a,b) + \frac{1}{Z(a,b)} + R(\Delta(b)) \right] g(b)db - kn \right\}.$$

Returning to firms reallocation decisions, there are two externalities created from firms to the representative household. First, the dispersion of markups between firms creates a misallocation of labor. This misallocation reduces he productivity of labor; Second, the firms do not internalize the benefit of matching transferable capital towards better firms. The second externality is a novel insight from our paper. It is natural the equilibrium is not efficient, which creates room for policy interventions.

We wait to discuss optimal policies until after we have set up the planner solution in Section 4. However, the analysis so far hints at the main role of policy. A policy that aims to improve efficiency should induce firms to internalize the externalities created towards households. One of the most natural mechanisms to reach this goal is competition from other firms in the marketplace, who can discipline the duopolists. In the quantitative model, we enrich the model with more realistic features in competition and endogenous entry.

2.3 Analytical Case: Role of Brand Reallocation

In this section, we consider a simple case of the model to highlight the role of brand reallocation on welfare and the sources of distortions. We will proceed by characterizing the endogenous objects analytically. Two simplifying assumptions are made: (1). we assume there are no fringe producers (entry cost $\kappa = \infty$) and (2) we assume the brand diffusion rate across duopolists is zero ($\chi = 0$). Under these conditions, the following lemma holds:

Lemma 1 *The target of reallocation, $B(b)$, is greater than b for any $b > -a$. For $b < -a$, the target of reallocation $B(b)$ is less than b.*

Proof. See Appendix. □

When $b > -a$, e.g. $z > 0$, brands reallocate towards the more efficient firm, we denote this type of reallocation as *efficient reallocation*; When $b < -a$, e.g. $z < 0$, brands reallocate towards the less efficient firm, we denote this type of reallocation as *strategic reallocation*. Why would firms reallocate the transferable asset towards ineffcient use, when the firm productivity is a complement to the brand? The economics comes from the increasing returns created by the variable markup. Bigger firms are able to charge a higher markup, and thus an additional unit of brand capital is worth more to them. Large firms, for instance, have a larger incentive to control a brand in their market than a similar firm of a smaller size. As an analytical example, we further focus on the cost function:

$$R(x) = \begin{cases} 0, & \text{if } x \leq \Delta \\ \infty, & \text{if } x > \Delta \end{cases} \tag{9}$$

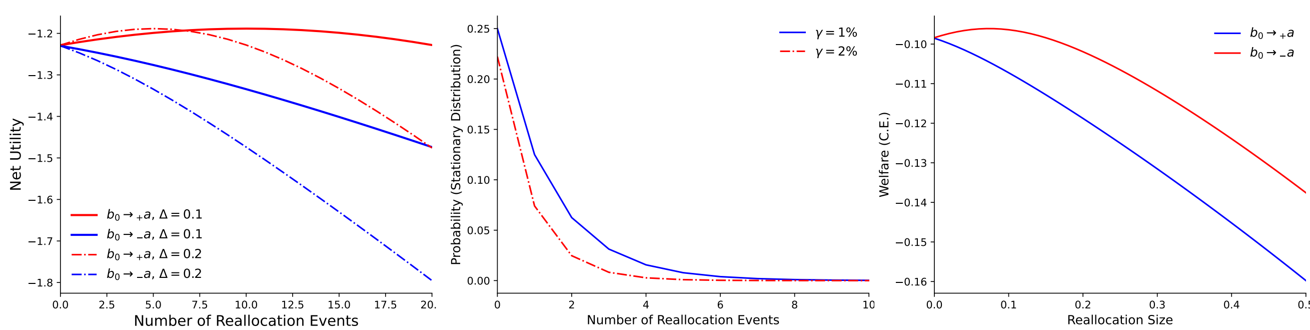
Lemma 2 *Assume $\chi = 0$ and the function form in (9), the distribution of product groups with respect to b has a probability mass function at $\{b_i\}_{i=0}^{\infty}$ with probability $g_i = \frac{\lambda}{\gamma}\left(\frac{\lambda}{\lambda+\gamma}\right)^{i-1}$, where $b_i = b_0 + i\Delta$ if $b_0 > -a$ and $b_i = b_0 - i\Delta$ if $b_0 < -a$.*

Proof. See Appendix. □

In this analytical case, the variation across product groups comes primarily from how many times a group has experienced a brand reallocation opportunity before the next creative destruction. Across product groups, this delivers a geometrically distributed probability mass function of brand gaps. As a result, the model generates a form of path dependence. The starting brand gap b_0 is crucial for determining the flows. When the initial brand gap is small enough, such that the follower has a market share larger than α (? this was 12), brands are always mismatched with productivity; When the initial brand gap is large enough, brands are always matched with productivity. These two types of distributions are observationally equivalent if we only observe the market shares. However, the economic and welfare implications are starkly different.

We explore this dynamic in Figure 1. This panel presents three graphs that summarize the main message of the analytical dynamics. First, Figure 1a reports the relationship between the number of reallocation opportunities and total welfare (net utility) in the economy. Figure 1b reports the distribution of the economy as a result of the number of reallocation opportunities before reset. Finally, Figure 1c reports the relationship between the amount possible to reallocate in one event (e.g., determined by policy or ownership frictions) and overall welfare.

Figure 1: Analytical Case: Utility and Distribution



(a) Utility and Reallocation (b) Distribution (c) Size of Reallocation

Notes: Analytical cases plots the utility (panel A+C), and probability distribution of states (panel B) against reallocation events (panel A and B) and Reallocation size (panel C). *Source*: author calculations.

Figure 1a illustrates the role of path dependency on welfare: small deviations in the initial brand capital (whether at the incumbent–blue or more productive entrant–red) are essential for brand capital flows and welfare. This is a core message of the analytical case and is studied further quantitatively. Figure 1b shows that this distribution is dependent on the degree of creative destruction, which we take as exogenous. The more creative destruction, the fewer reallocation events. Figure 1c shows how much the amount of brand capital reallocated can also shift welfare. Overall, this case shows the importance of understanding initial conditions, creative destruction, and the nature of brand capital transactions for understanding optimal policy and welfare.

The simple model enables us to link firms' activities in both sales to consumers and in trading brand capital to aggregate outcomes in the macroeconomy. This stylized model will be expanded in Section 4 to embed more empirical realism, but the main idea of linking firm productivity and brand capital to aggregate outcomes will maintain the spirit of this framework.

3 Data and Empirical Analysis

Our theory treats brand capital and core firm productivity as the joint determinants of market share. Empirically, there is a rich literature on production functions and patenting as ways to measure firm productivity. The literature has been mostly silent on proper proxies for brand capital. This section introduces a crosswalk between firms' trademark holdings and brand-level data on prices, customers, and revenues to provide a foundation for analyzing brand capital and its interaction with fundamental firm characteristics.

We start by discussing the data construction, as we link USPTO trademark data to information at the retail level on firm prices and sales. We then use the constructed dataset to illustrate core facts on brands and brand reallocation. The event studies, where brand ownership is reallocated across firms, provide a key ingredient in the decomposition of a firm and brand effect on market share. We bring the data directly to the model in the quantitative analysis in Section 4.

3.1 Data Construction

The first unique data contribution of this paper is the merge of US Patent and Trademark Office (USPTO) trademark data with RMS Nielsen scanner data. The pairing provides details on brand history, including the prices, sales, and age of each brand, and firm-specific features, such as firm sales revenue and brand holdings. The merged dataset admits exploration of the mechanics of brand introduction, brand development, and brand reallocation.

USPTO trademark data provide a unique and comprehensive insight into the distribution and history of brands across firms. Individuals or firms apply for trademarks when they want legal protection of their brand capital. The USPTO defines a trademark as "any word, phrase, symbol, design, or a combination of these things that identifies your goods or services. It's how customers recognize you in the marketplace and distinguish you from your competitors." More firms participate in trademarking than patenting, and there are more trademarks than patents reallocated across firms each year. Given our focus on reallocation, we restrict our attention to reassignments and mergers[3].

To enable the study of price and sales data, we employ detailed bar-code level data from Kilts-Nielsen Retail Measurement Services Data from the University of Chicago Booth School of Business. The data are large and comprehensive in the consumer product space. This dataset delivers significant coverage for products, brands, and firms, which we detail in Appendix A and is discussed in Argente et al. (2020b).

[3]There are multiple types of transactions, which we discuss in detail in Appendix B.4.

One point of departure from the literature is our focus on brands rather than products. A product contains a specific 12-digit identifier, which may contain slight product variations under a broader brand (e.g., size differences, new editions, seasonal variations). Given our interest is primarily linked to the customer association with the product, we focus on a higher level of aggregation. This also makes the merge to USPTO trademark data easier.

We perform the merge by focusing on the brand and the firm as a pair. We employ a fuzzy merge to connect brand names in RMS Nielsen scanner data to USPTO trademark data. Whereas this merge is the first we know of that links USPTO *trademark* data to Nielsen scanner data, Argente et al. (2020a) link USPTO *patent* data to RMS Nielsen data. More trademarking firms and brands are matched in our sample, likely due to the different nature of patents and trademarks. In particular, we are able to identify all products connected to their brand name as long as the trademarked brand name is similar enough to the brand name on the product in the store, and an active firm holds it in the dataset at a point in time. We expand on the details of our merge in Appendix A. Table 1 details the merge on the trademark side and the RMS Nielsen side.

Table 1: Summary Statistics on Trademark–Nielsen Merge

	Unique Count	Years Active	Share Match (%)
USPTO Trademark Data			
Brands	5.36M	1870–2020	1.9%
Firms	371,021	1870–2020	15%
Canceled Brands	2.12M	1970–2020	
Transactions	915,076	1970–2020	
RMS Nielsen Scanner Data			
Products $\times$ Group	1.64M	2006–2018	
Brands $\times$ Group	82,525	2006–2018	57%
Firms	23,232	2006–2018	54%
Brand $\times$ sales		2006–2018	82%

Notes: Summary statistics on share of merge brands in both datasets. Source: USPTO Trademark Data and RMS Nielsen Scanner Data.

We stress a few points from Table 1. First, when we merge brands weighted by sales, we capture 82% of sales in the data. Without sales weights, we capture fewer brands (57%). Some small firms may choose not to protect their intellectual property via legal means. Second, many trademarks are not associated with consumer packaged goods, so a smaller share of trademarks are merged. Third, multiple brands are associated with a single firm on average, in line with the model; furthermore, multiple UPC products are connected to a single brand. On average, we observe 9 unique UPC products per branded product. This connects to our framework where the brand is a capital good at the firm that provides a family of products an umbrella by which they access the customer.

3.2 Empirics of Brands and Brand Reallocation

We start our empirical analysis with a study of brands mirroring the main features of the model. One mechanism we propose is the potential mismatch between productivity and brand capital. Large firms may accumulate customer capital and corner the market rather than sell their brand capital to more productive firms. Brand reallocation thus enables this large unproductive incumbent to persist. On the other hand, brand reallocation can also generate efficiency gains through matching productive firms with customer access. This mechanism can be studied in the data.

We focus on the role of brands as transferable customer capital and use this framework to study the evolution of market share and the fixed and transferable components of firms. We start by breaking down the contribution of brand creation, maturity, and reallocation to firm growth and decline. We then evaluate the transfer of brand ownership across firms, which provides insights into both the outcomes of brand reallocation and enables the decomposition of the brand and firm components for the main drivers of market share.

Overall, we find that brand reallocation plays an important role in market share dispersion, especially for large firms. We then study brand reallocation events. On average, brand reallocation leads to increases in sales (expanding customer base, an efficient outcome) and an increase in prices (an increase in the implied markup, an inefficient outcome). These reallocation events mask rich heterogeneity that we discuss briefly in this section and discuss in the quantification in Section 4.

Firm-level characteristics are the most common framework in the literature for explaining concentration and market share (Edmond et al., 2015; Akcigit and Ates, 2021). The ability to observe brands move across firms provides insights into how much market share outcomes come from the firm versus the brand level. The event studies illustrate that firm components do matter, as we see brand sales move after an event, but also exhibit aspects of persistence at the brand level, as customers do not exit the brand at a higher rate. In Appendix A, we show this in greater detail in an exercise following the labor market literature (Abowd et al., 1999; Bonhomme et al., 2019).

In M&A literature, there is missing data on being able to identify revenue in the acquired firm. With patent data (e.g., Akcigit et al., 2016), one can observe the technology transfer but no outcomes directly linked to revenue, prices, or market share. Thus, tracking brands in event studies provides a unique way to measure the firm and brand-level effects. In our event studies, we are able to continue to track brands throughout their life cycle with the corresponding revenue attached to them.

The Drivers of Market Share. The development of market share is a dynamic process with the role of customer base at the center. Even the largest firms took significant time to build their customer base. In building market share, a classic tradeoff for firms is the choice between internal development and acquisition. Empirically, we bring this framework to the realm of customers, where there is a growing body of evidence that the development of customer base is at the center of firm growth (as noted by Einav et al., 2021 and Afrouzi et al., 2023).

In the following empirical exercise, we decompose the driving forces behind changes in firm market share in terms of brands. Embedding the classic tradeoff in the theory of the firm into branding, we focus on the relative roles of brand creation (new brands), brand maturity or advertisement (changes in the share of old brands), and brand reallocation across firms. We link these three components to the changes in market share within a given product group (e.g., "SOFT DRINKS"). We ask how much the variance of firm market share growth is driven by these three forces. For brand creation, we include brand introduction (the first year of a brand) and brand death (removal of brands from the market). For brand maturity, we include all growth and decay from brands held by their parent firm in two consecutive periods. For brand reallocation, we focus on the net transacted brands in terms of market share. For instance, if a firm sells a brand this counts negatively in terms of the market share lost, whereas if a firm acquires a brand, it counts positively in terms of market share gained.

We focus on a variance decomposition of the change in firm market share due to the three forces. Table 2 reports the total variance (first column) and the share explained by the three components above.

Table 2: Variance Decomposition of Firm Market Share Growth, Weighted by Group Size

	Total Variance	New Brands	Incumbent Growth	**Reallocation**
All Firms	0.72	6%	86%	3%
Firms >0.01% Share	0.24	7%	81%	10%
Firms >0.1% Share	0.14	7%	72%	21%
Firms >1% Share	0.07	5%	44%	**50%**
Firms >5% Share	0.04	1%	19%	78%

There are multiple takeaways worth stressing from Table 2. When there are no firm cutoffs for size the total variance of firm growth is large, due to many small firms, and the share of firm growth variance driven by reallocation is small. Some very small firms may simply enter and exit and not trade brand capital or achieve interest from acquiring firms. However, even when the cutoff for firm size is simply increased to 0.01% of the total market share in a given product group, the role of reallocation starts to become a major driver of market share, more important than new brand creation. For firms larger than 1% of the market, reallocation plays a more important role than maturity and creation, indicating its central role in the market share growth of large firms.

We leave out the covariance terms in this table for parsimony. Overall, covariances explain 5% or less of overall variance, indicating most of the variance in market share comes from the three sources of market share growth and decay.

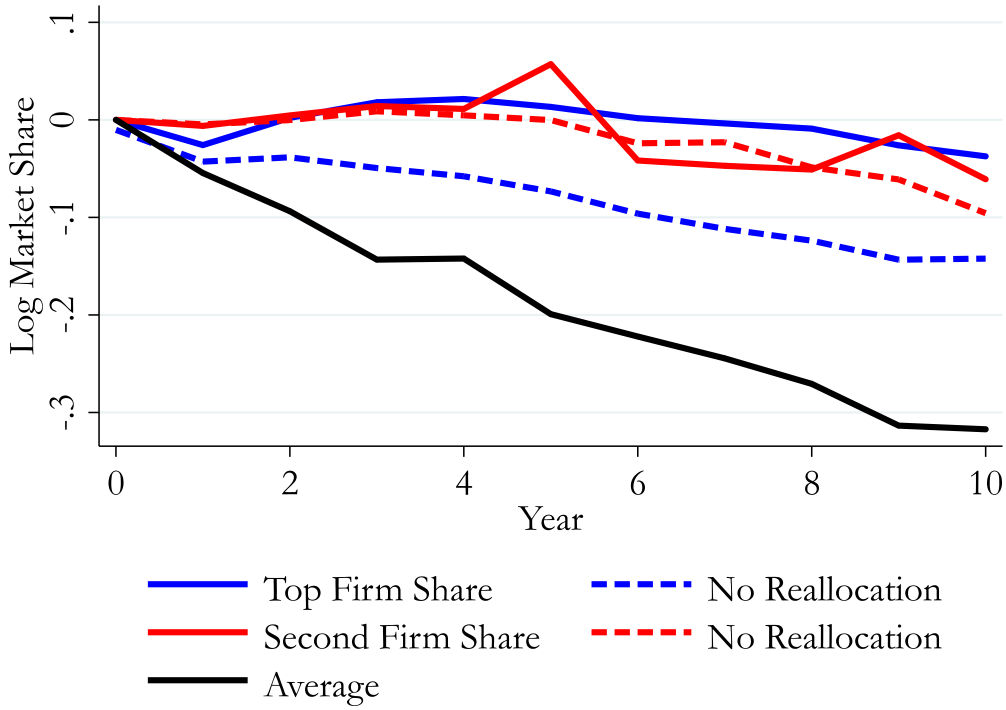
Figure 2 zooms in on the top two firms in each product group and evaluates the evolution of their market shares over ten years. To understand the evolution of firm market share and the role of reallocation, we plot the evolution of the leader's share, follower's share, and average firm's share over time. We normalize the values to be comparable in the initial period. For the top two firms, we plot the evolution of their log market share (solid lines) and the counterfactual market share in a market without brand reallocation (dotted lines). We report this evolution over a 10-year period.

Figure 2 highlights a couple of key facts that cohere with our theoretical model. First, market share leadership is persistent. The gap between the top two firms and the average firm grows over time. After 10 years, both leader and follower show a similar market share to the beginning period, while the average firm has 30% less market share.

Second, the reallocation of brand capital explains a large share of this persistence, in particular for the top firm. Without brand reallocation to the large firm, the change in the gap of the log between the leader and the average firm would shrink by almost 40%. In constrast, the second firm mostly relies more than the leader on its existing brands and the introduction of new brands. While the second firm still has some gap at the end of the period, the gap is much smaller than the leading firm (around 2% difference versus 12% difference).

Figure 2 indicates acquiring brands is an important strategy for large firms to keep their advantages

Figure 2: The Evolution of Market Share



Notes: Top two firms determined by average market share, logged and initialized at period 0. The dotted lines remove all net inflows/outflows from each firm contributing to market share. Shares are weighted by product group size in sales. Source: USPTO/RMS Nielsen.

with respect to small firms and an important driver of the persistence of concentration in the product market. Interpreting the brand reallocation events through the lens of our model, they also provide a unique window to look into the relationship between firm core productivity and tradable brand capital. We turn to this next.

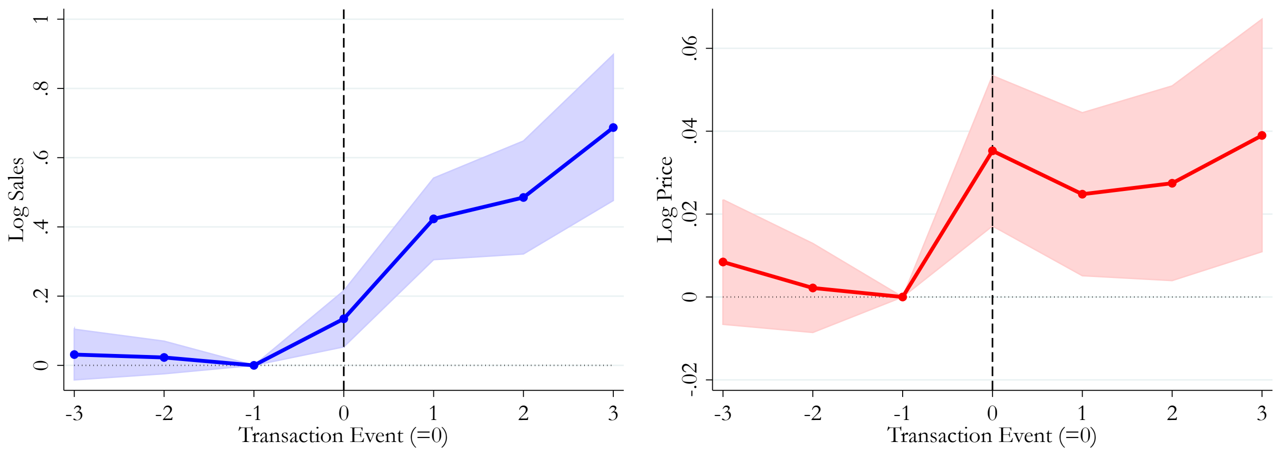
Brand Reallocation Events. We focus on two different outcome variables in our event studies: revenues and prices. We compare the reallocated brands to a group of similar brands within the same product group around the time of the reallocation event. These similar brands will be comparable brands that did not experience a reallocation event. We then estimate the following regression in Equation (10),

$$\log y_{ikt} = \sum_{\tau=-3}^{3} \zeta_\tau \times \mathbb{I}\{t=\tau\} + \sum_{\tau=-3}^{3} \alpha_\tau \times \textit{reallocated} \times \mathbb{I}\{t=\tau\} + \xi_t + \theta_{ik} + \epsilon_{ikt}, \tag{10}$$

where the unit of analysis is at brand i within product group k in the year t. In equation (10), y_{ikt} is either the total revenue or the average price. We define the average price of a brand-group-year cell by aggregating all transactions within this cell by taking the sales-weighted geometric average of transaction-level prices. *reallocated* is an indicator variable on whether the brand belongs to the reallocated group. ξ_t is a year fixed effect; θ_{ik} is a brand-group fixed effect. The coefficients $\{\alpha_\tau\}_{\tau=-3}^{3}$ is particularly interesting to us. They estimate the average difference of outcomes for the reallocated brands compared to the similar brands, as selected by the coarsened exact matching algorithm. We match on pre-event sales and product group categories and year, applying weights to generate a synthetic control, as discussed in Blackwell et al. (2009).

Figure 3 plots the estimated $\{\alpha_\tau\}_{\tau=-3}^{3}$ for the regressions for sales (panel A) and the regression for prices (panel B).

Figure 3: Brand Transactions and Matched Pairs



(a) Sales × Transaction

(b) Prices × Transaction

Notes: Coarsened exact match coefficients. Match is conditional on brands opening with at least $10,000 in sales. Match is made on pre-trend sales, exact product group, and exact year. 95% confidence interval standard errors clustered at the brand-group level.

Table 3: Event Breakdown

	Avg Revenue	Avg Price
Baseline	0.43	0.032
Weighted	0.13	0.010
Select on Big Buyers	1.88	0.049

Notes: Baseline selects cutoff at $10,000 in initial sales. Weighted version and big buyers takes all variables. Big Buyer is defined as the top 10% of buyer market share which is approximately 0.08.

We discuss two main takeaways from Figure 3. First, the reallocated brands experience an increase in sales relative to their unreallocated counterparts. After 3 periods, sales are up around 60% from the pre-transaction date compared to other similar brands in the same product group, averaging 43% over those post-event periods. This indicates an expansion of revenue for the reallocated brands. Interpreting these results from the lens of our model, we argue that the average reallocation events are towards more efficient producers (the frontierrs within their groups).

Second, there is an increase in prices of the reallocated brands compared to their counterparts. The expansion in sales does not happen alongside a decrease in prices. On its own, an increase in prices could suggest a less efficient acquiring firm or higher markups. Given the increase in sales, buying firms appear to exhibit advantages relative to the selling firm. Interpreting these results from the lens of our model, we argue that markup differences between the selling and buying firms play a role. Although the average reallocation is towards more efficient firms, it is also towards the firms with higher markups. The markup effect outweighs the efficiency gain for the average reallocation event.

Even if the acquiring firm were significantly more productive on the supply side, it appears the gains are not present to the consumer due to an increase in markup. The distinct outcomes at the sales and prices level suggest two important mechanisms through the lens of our framework. First, brands appear to be reallocated to firms who can expand sales relative to the existing incumbents. Second, there is tension in this process, as acquiring firms both expand sales and raise prices. To understand more detail on these events, we turn to the role of heterogeneity, market access, and local market effects.

Heterogeneity. While the average sales and price effects indicate an interesting tension between productive and strategic interaction, it masks a significant heterogeneity across transactions. 62% of transactions experience an increase in sales upon transaction relative to control brands, whereas 51% of transactions experience an increase in prices. This heterogeneity immediately indicates the importance of taking into account the context of a brand reallocation to understand the effects. In the appendix, we break this down further by extracting the brand and firm component of the variation in sales adapting a method from Bonhomme et al. (2019).

Part of the motivating model presented indicated that depending on features of the market structure, such as productivity and the market share gap, some transactions may be efficient while others may be strategic. This tension can be found in the data, as some transactions exhibit what looks like purely a productivity gain (only sales go up, and prices stay flat or decline). In contrast, others exhibit a purely strategic effect (prices go up, with a negative effect on sales). 29% of transactions exhibit a price change below the average and a sales change above the comparison group (primarily productive effect). 17% of transactions exhibit a sales change below the average and a price change above the comparison group (primarily strategic effect). We revisit these details in Section 4, where we quantify our model.

Geographical Expansion. To expand on the nature of reallocation, we aim to understand which margins of sales expand after a brand reallocation event. We find that acquiring firms are more likely to expand the brand into different retail establishments, consistent with the acquiring firms expanding customer base. We do similar event studies to show this retail expansion. Appendix B details the empirical strategy in depth.

Local Market Effects. To further understand the interaction between market share and brand reallocation, we look at local markets or specific retailer markets. In the model, we find that firms with a larger presence in product markets have higher markups. We study this empirically through the lens of our events: how does the change in market share interact with a firm's pricing decision after a reallocation event? The model predicts that when firms have larger market share, the firms' markups will be higher. We study this in the context of firms with different market share across retailers.

In order to study this, we look at the change in the prices at the brand-parent store level in order to understand how market share will predict changes in prices. We evaluate the how the price at the brand-level changes when a brand is reallocated across firms, given the change in firm shares observed during this reallocation.

$$\Delta y_{ijt} = \alpha + \beta \Delta firm\_share_{j,j',t-1} + \lambda_t + \xi_j + \epsilon_{ijt}$$

Table 4: Local Firm Market Share and Change in Prices and Sales

	(1)	(2)	(3)	(4)
	Δ Log Price	Δ Log Price	Δ Log Sales	Δ Log Sales
Δ Firm Market Share	1.19***	1.53***	-2.07	-1.21
	(0.000)	(0.000)	(0.074)	(0.269)
Sales-Weights	No	Yes	No	Yes
Fixed Effects	Firm and Year	Firm and Year	Firm and Year	Firm and Year
N	10700	10683	10700	10683

p-values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

We find that prices tend to move positively with firm market share, consistent with firms having larger market share putting more upward pressure on prices. When it comes to sales, we don't find significant effects, indicating a potential tension as large firms have less incentive to expand sales but more capability to do so.

4 Quantitative Analysis

In this section, we describe the estimation of parameter values, quantify the source of market concentration, and perform the counterfactuals regarding policy implications.

4.1 Estimation

We estimate the set of parameter values that brings the model-predicted patterns close to the empirical patterns of brand reallocation. We start by assuming the economy is in its steady state. The following parameters are crucial for our quantitative analysis: (a). the creative destruction process, which involves the creative destruction rate γ, the frontier firm's advantage a; (b). the cost of brand reallocation, which we parameterize to be $R(\Delta) = r_0 \Delta^{1+\frac{1}{r}}$. We will refer to r as the reallocation elasticity and r_0 as the cost shifter of reallocation; (c). the exogenous brand diffusion rate χ; (d). the entry cost κ.

Externally Calibrated Parameters. We calibrate a subset of parameters directly to the values from the literature. The model is calibrated to an annual frequency, so we set the discount rate $\rho = 0.03$. We calibrate the substitution elasticity $\sigma = 3.9$. This value is the median across-firm substitution elasticity from Hottman et al. (2016), estimated from a similar demand system and RMS Nielsen Scanner data. This number, together with the Cobb-Douglas aggregator across product groups, implies the minimum profit margin of firms is around 0.25, and the maximum is 1.0. This substitution elasticity has important implications for our counterfactuals and we report the quantitative analysis given different values of substitution elasticity.

Creative Destruction Parameters. The brand reallocation events provide a window to separate firm productivity difference a from the brand gap b. We estimate a from the following steps. First, we set Poisson rate at which the creative destruction events arrive to $\gamma = 0.02$, which is the value from the growth literature. Second, we estimate (χ, a) to match the average estimates from the brand reallocation event study.

The reduced-form estimates on sales and prices from the event studies include changes in appeal, productivity, and markup from the lens of our model. We utilize the model-implied equations to back out these changes. From the model, when a brand that changes hands, its sales change by:

$$\log \text{Sales}_{it+1} - \log \text{Sales}_{it} = a_{k(i)t} + (\sigma - 1) \log \frac{\mu_{J(i,t+1)}}{\mu_{J(i,t)}},$$

The model provides a formula for markups at the firm level. Thus, we use the pre-reallocation market shares at the selling and buying firms to impute the change in markups. The residual change in revenues provides a distribution of a. This distribution could directly inform the evolution of a if brand reallocation events always went to the most productive firm. However, as in our model, these events can go in the opposite direction of the productive firm. We thus jointly calibrate (a, χ) to match the average effect $\hat{a}$ and the share of events that lead to positive $\hat{a}_i$.

Reallocation Cost. We calibrate the reallocation elasticity r to match the size-reallocation profile observed in the data. Because of the selection in reallocation events, we need to jointly estimate the brand reallocation with other parameters. Here, we make a heuristic argument for why the size-reallocation profile is informative for the reallocation elasticity. From the model, the reallocation size Δ is linked to the size gap between the two firms by the following first-order condition:

Entry Cost. The entry cost is calibrated to match the average top-two shares. In the data, this share is 45%. Through the lens of our model, this implies an entry cost of $\kappa = 0.32$.

We rely on the optimality conditions for innovation, entry, and acquisition to recover these parameters. First, we note the marginal value of the state variables can be written as functions of the quality gap ϕ and growth rate g. Both variables have data counterparts. Specifically, the quality gap ϕ has a one-to-one mapping to the observed market share given σ_k; the growth rate g is linked to the brand creation rate by the fringe firms. With these two variables, we can directly calculate the marginal value of brands for the group leader. For each product group, we find the set of parameters (κ_k^e, κ_k^s) that minimize the distance between the data and the model's prediction of the leader's innovation rate, average selling rate, and innovation rate of fringe firms.

Table 5: Estimation Moments and Parameters

Parameter		Value	Moment	Data	Model
Independently Calibrated					
Discount Rate	ρ	0.03	Annual Risk-free Rate	**Exact Match**	
Substitution Elasticity	σ	3.90	Hottman Redding Weinstein (2016)		
Jointly Estimated					
Brand Volatility	ν	0.30	Residual volatility firm growth		
Creative Destruction					
Rate	γ	0.04	Growth Rate	2.31	2.31
Step	a	0.72	Sales Event Study	0.36	0.36
Initial Brand Gap	b_0	-1.10	Shr. Pos Events	0.69	0.69
Reallocation Cost					
Level	r_0	0.64	Average Reallocation Size	2.31 (p.p)	2.31 (p.p)
Elasticity	r	0.18	Reallocation-Size Profile	0.39	0.39
Fringe Entry Cost	κ	0.48	Top-2 Share	0.45	0.45

Notes: Parameters estimated separately (top panel) and jointly (bottom panel). Source: RMS Nielsen, USPTO and author calculations.

Estimation of Matching Elasticity. We estimate the innovation and matching elasticity using indirect inference. The targeted moment for this elasticity is the age profile of a brand getting transacted. In our model, the difference between the transaction rate for a new brand and for a mature brand is governed by the difference in marginal benefits and the matching elasticity. The difference is in the matching elasticity. In the extreme, if the matching function is inelastic with respect to tightness, no differential in the sales-transaction rates exists. Our estimation yields a matching elasticity of 0.292.

4.2 Brand Reallocation Event: Productive and Strategic

We consider specific events to ask, what happens to prices and sales when a brand is reallocated? We consider a specific event and utilize the changes in prices and sales to understand the distribution of the exogenous productivity parameters α_C and α_L.

To hit the frequency of reallocation, we study the relative fraction of transactions of each type of brand. We study the cost of reallocation by leveraging how much brand capital moves with the log of the gap between the firms.

We use the literature for the elasticity of substitution across and within product groups. To calibrate the entry elasticity, we use recent work from Klenow and Li (2022). For the reallocation arrival rate (λ), we use the annual number of trades. To measure the reallocation elasticity, we use the volume-size correlation.

We then jointly estimate the reallocation cost, volatility of the shock, and the operation costs from the overall shares and average size of the brand exchange.

We can then characterize the amount of brand mismatch. Overall, 23% of the market has brand reallocation mismatched to productivity, while 69% of the market looks "productive and strategic," meaning that acquiring firms use acquisitions to raise markups. Only 8% of the markets are purely efficient.

5 Policy Implications

In this section, we perform the counterfactual analysis of different policy instruments. As a benchmark, we start by setting up the planner's problem in our economy. We show that the gap between the decentralized equilibrium and the planner's allocation reflects the static markup distortion and the dynamic misallocation between brand capital and firms' core productivity.

5.1 Planner Solution

The planner makes both static decisions and dynamic decisions to maximize the representative household's discounted utility. Statically, she chooses (a) how much firms produce given their productivity and their brand capital and (b) how many fringe firms enter. Dynamically, she chooses how much brand capital to reallocate across firms whenever the reallocation events arrive.

The static allocation of production and entry is standard. The planner chooses the production proportional to the productivity of firms. The optimal allocation of production and entry is discussed in detail in the Appendix, and we directly write out the optimal labor productivity as $z^*(b) = \frac{e^{a+b}+1+n}{e^b+1+n}$. With this notation, the consumption in a product group with state b is $c^*(b) = z^*(b)l(b)$, where $l(b)$ is the amount of production labor. The optimal number of fringe firms, according to the planner is:

$$\frac{1}{\sigma-1}\int_{-\infty}^{\infty}\frac{1}{e^{a+b}+1+n}g(b)db = \kappa. \tag{11}$$

The planner equalizes the marginal benefit of creating a fringe product to the marginal cost. The marginal benefit, conditional on the planner choosing the optimal production, reflects purely the love of variety from the new products. We thus write the net consumption at a product group with state b as $c^*(b) = \frac{1}{\sigma-1}\log\left(\frac{1+e^{a+b}}{1+e^b}+n\right)-1$.

We now write out the planner's dynamic decision. The planner chooses $B(b)$ to optimize the following problem:

$$\max_{B(b)} \int_0^{\infty}\int_{-\infty}^{\infty} e^{-\rho t}c^*(b)g(b)db, \tag{12}$$

s.t.

$$\dot{g}(b) = -(\lambda+\gamma)g(b) - \chi g'(b) + \lambda\int_{-\infty}^{\infty}\mathbb{I}\{B(b') = b\}g(b')db'.$$

This dynamic decision has a HJB equation resembling the equilibrium's joint surplus equation:

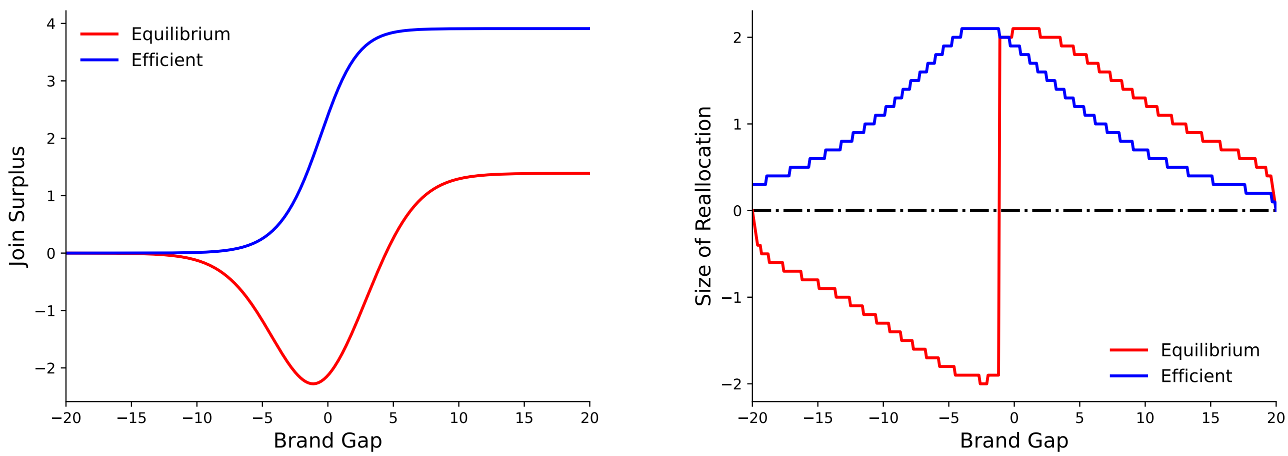
$$(\rho+\lambda+\gamma)\omega^*(b) = c^*(b) + \max_B \lambda\left(\omega^*(B) - R(|B-b|)\right) + \gamma\omega^*(-\infty) + \chi\omega^{*\prime}(b). \tag{13}$$

We now turn to Figure 4. This figure demonstrates the efficient and equilibrium allocation (panel 4a) and the reallocation decisions of the planner and the firms in equilibrium (panel 4b). We plot the equilibrium (red) and the efficient (blue) surplus in panel A, and reallocation size in panel B.

We stress a few takeaways from this figure. First, Figure 4a shows the difference between the planner's surplus and the equilibrium surplus. When the brand gap is low, the firms prefer consolidation as it enables a larger profit margin. This, however, keeps brand capital in the inefficient vintage firm. The planner wants the more productive firm to hold the brand capital, so the planner gains from increasing b. This comes from sorting the brand capital to the more productive firm.

Figure 4b illustrates the reallocation decisions in equilibrium and for the planner. First, the planner always has incentive to reallocate to the frontier firm (blue line). However, when brand capital is sorted to the vintage firm ($b < -a$), the firms in equilibrium sort brand capital to the inefficient leader to again

Figure 4: Comparison between Equilibrium and Planner Allocation



(a) Joint Surplus (b) Reallocation Policy Function

Notes: The left panel plots the joint surplus according to planner and equilibrium allocation, according to the baseline calibration. The values are normalized by subtracting the level at $b = -20$; The right panel plots the corresponding reallocation policy function.

consolidate the profit margin. Once brand capital crosses the productivity threshold ($b > -a$), the planner and equilibrium have similar incentive to consolidate brand capital to the more productive firm. Overall, these figures illustrate the distinct strategies of the firms in equilibrium and the planner in particular when brand capital is sorted to the vintage, less productive, firm.

5.2 Welfare Implication of Alternative Policies

Once brand capital and productivity are considered to jointly determine market share, classic policies under monopoly have different implications. We compare the planner solution to two alternative policies: *size-based subsidies* and *entry subsidies*. These two policy instruments are discussed substantially in the literature as remedies for the distortions from firm markups. Due to the disconnect between market leadership and productivity, our quantitative analysis shows that a blunt size-based subsidy could backfire.

Size-based policy. With a size-based policy, we consider a subsidy that implements the static optimal allocation when the policy maker only has data regarding the market shares, but not (a, b) separately. Following Edmond et al. (2023), we derive the optimal subsidy schedule such that firms produce according to $z = \frac{1+e^{a+b}}{1+e^b}$. Under the subsidy, firms have a joint surplus:

$$\omega_{emx}(b) = \frac{\sigma}{\sigma - 1}\Big[2\log(z(b) + n) - \log(z1 + n) - \log(z_2 + n)\Big] - \frac{z_1 + z_2}{z_1 + z_2 + n}.$$

Entry Subsidy. With an entry subsidy, we consider a subsidy on the fringe firm entry cost that maximizes the steady-state welfare of the representative household.

Results. Table 6 reports the productivity losses in three environments: The first environment is the average case in our sample Hottman et al. (2016), which takes the average elasticity of substitution from their paper. We add a "high-markup" world ($\sigma = 2$) and a "low-markup" environment ($\sigma = 8$), and compare overall welfare under the efficient scenario to the size-based policy and the entry policy. Consumption equivalent welfare is made up of consumption, labor in production, labor in entry, and reallocation labor.

Table 6: Welfare Under Alternative Policies

	Efficient	Size Only	Entry Subsidy
Baseline $\sigma = 3.9$			
Consumption, $\log C$	59.0	58.0	5.7
- Product Labor, L_P	41.0	41.0	1.8
- Entry Labor, L_E	-12.0	-12.0	3.6
- Reallocation Labor, L_R	0.086	30.0	-0.0013
Welfare (C.E.)	**30.0**	**-1.9**	**0.27**
High Markup $\sigma = 2.0$			
Consumption, $\log C$	97.0	91.0	58.0
- Product Labor, L_P	65.0	65.0	7.2
- Entry Labor, L_E	-18.0	-18.0	34.0
- Reallocation Labor, L_R	0.35	57.0	-0.0092
Welfare (C.E.)	**49.0**	**-14.0**	**17.0**
Low Markup $\sigma = 8.0$			
Consumption, $\log C$	36.0	35.0	-2.7
- Product Labor, L_P	24.0	24.0	-1.2
- Entry Labor, L_E	-7.8	-7.8	-1.7
- Reallocation Labor, L_R	0.046	26.0	0.0017
Welfare (C.E.)	**20.0**	**-6.2**	**0.17**

Table 6 presents some core messages of our paper. We start by focusing on our baseline case. First, we note that the equilibrium diverges significantly (30% in welfare equivalence) from the efficient outcome. This comes both from the classic markup distortion but also from the mismatch between brand capital and productivity. Second, the size-based subsidy, which would restore efficiency in a model without brand capital, creates welfare losses (-1.9%). This primarily comes from the inducement of firms to reallocate brand capital to the less productive vintage firm. The efficient outcome will always push brand capital towards the more productive firm even if small, avoiding this problem. Finally, an entry subsidy can have modest positive effects (0.3%).

The elasticity of substitution has important implications for policy counterfactuals. We show that changing the elasticity of substitution does not change the qualitative message of the first panel but changes some quantitative results. In both the low-markup and high-markup environment, we find the large divergence in welfare from the optimal and the equilibrium, the size-based subsidy backfires, while entry policy can have positive effects. Entry subsidies are especially strong in high-markup environments, where entrants reduce the market power of incumbents and induce efficient reallocation.

To explore the differences in welfare counterfactuals depending on the heterogeneity, we take two other cases. The first represents the "low substitution - low entry cost" world. The second is the "high substitution - high entry cost" world. Both cases will be illustrative.

To understand the theoretical implications of our framework, we compare market-level outcomes to solutions to a social planner's problem. There are different ways to define a social planner's problem in our setting, and those definitions will help elucidate various mechanisms. At one end of the extreme is a social planner who can decide all allocations and production decisions. This social planner would internalize all market forces and allocate all market access to the most productive firm, inducing the firm to produce as if it was a small firm. Another planner could control entry and reallocation, but not production decisions. Finally, a more constrained planner may focus only on the reallocation decisions. These three characterizations can speak to various mechanisms at play.

We find that the planner who can control entry and reallocation moves the market about halfway to the first best solution, which plays to our mechanism as distinct from Edmond et al. (2023). In particular, entry and reallocation press on the sorting of good brands to good firms, whereas production decisions eliminate the costly markup. This speaks to a more general point: when a brand gap or customer capital gap is included, and is tradable, it can significantly alter the predictions on productivity. We use this framework to think through counterfactual policies that may have different implications from the literature.

The introduction of a brand gap or customer capital gap provides some intuitive adjustments to a standard model. Firms want to accumulate customer capital in a single firm to enable markups, which leads to a dynamic misallocation. Indeed, this dynamic misallocation can be very persistent but may be path-dependent. The key mechanism is that a firm that gets positive productivity shocks will expand its brand capital and start to acquire. Even as this firm gets negative productivity shocks, it can maintain its high brand capital through acquisition, which small firms prefer because the large firm shares the surplus extracted from the customer.

This core mechanism changes the nature of many policies. The goal of the social planner's problems is to think through feasible policies in the counterfactuals through the lens of market structure.

We turn to policies to further elucidate this mechanism and its distinction from the current state-of-the-art literature. To better understand the mechanics of our quantitative framework, we study a mix of policies that can speak to the role of brand and productivity gap in comparison to the macroeconomic literature.

We study an HHI policy that limits market concentration. We find that this policy has mixed effectiveness but is in fact more effective in our framework than in standard models. We take different thresholds to illustrate the effectiveness. When at 30%, HHI policy can have positive effects. The main reason for this that the limit on HHI kills the bad equilibrium where market access is sorted to the unproductive firm just for the purposes of concentration. By capping HHI, this not only limits a high markup equilibrium, it only limits the flows moving from more productive smaller firms to less productive larger firms.

However, the cost of limiting HHI is it limits sorting of market access and productivity to the better firm. When at 10%, we find that HHI policy does poorly: most sorting and market size is efficient, and limiting concentration to such a degree limits the efficient capacity of large firms. The question of the effectiveness of the policy in the general case has to take into account the gaps of the firms and the corresponding level of HHI caps.

We can say more about HHI: standard model says you want to ban M&A in market with HHI, here we point to misallocation effect. selection is high with size differential. When reallocation occurs between two equal-sized firms, the reallocations tend to be more efficient.

Second, we evaluate policies on entry. Entry policies can have countervailing effects. First, entry can boost growth on its own. We shut down this mechanism by playing into the rational inattention framework: entrants are simply trying to capture more customer attention. However, entry can still have positive and negative effects. The first question is related to the relative productivity of entrants. If entrants are less productice

Third, we evaluate taxes and subsidies on the transfer of intellectual property in trademarks. We find taxes in general are a poor policy tool. Part of the reason is many transactions in trademarks are in net efficient, and what authorities may want to cap is rather the large concentration brought about by many transactions, rather than taxing each transaction.

Overall, these policies provide more texture than standard models that focus on firm productivity gaps. Given that branding is essential to firm market share, it seems these features of market structure should be taken seriously by policymakers.

This section explores the main quantitative results of our framework. First, we discuss the welfare incidence of brand reallocation and the shares of productive and strategic reallocation across firms.

5.3 Endogenous Advertisement

In this section, we extend our baseline model to consider the role of endogenous advertisement. When firms can endogenously invest in their customer base, the rate at which brands diffuse differs under alternative policy regimes. This rate further interacts with the endogenous reallocation decisions. Additional distortions show up in an equilibrium with endogenous advertisement as well. First, frontier firms can be discouraged from investing in their customer base due to the discouragement effect from variable markups, as in Aghion et al. (2005); Secondly, firms can invest excessively in their customer base due to the zero-sum feature of brand reallocation.

Model of Advertisement Investment. This extension is identical to our baseline model, with two additional assumptions. First, firms can choose the rate at which the brand gap evolves. The frontier firm can increase the brand gap by η, by paying cost $D(\eta)$. Symmetrically, the vintage firm can decrease the brand gap by ψ, by paying cost $D(\psi)$. We assume D is an increasing and convex function. Coupled with the exogenous brand diffusion rate χ, the evolution of brand capital follows: $\dot{b} = \eta - \psi + \chi$. Because the advertising choices involve the value of duopolists separately, the bargaining power of them matters. We denote the bargaining power of the frontier firm to be ϕ. The results are reported under different bargaining powers.

We highlight some main outcomes on the advertising framework. Firms endogenously choose advertising which can help match consumers to the better product. In a world with close competition, the more productive firm will advertise with greater intensity (insert equation) than the less productive firm, growing their brand capital and matching with consumers.

This outcome is intuitive: when firms have an opportunity to deliver productivity to the masses, they will advertise more to raise more awareness. However, the role of market structure is essential for firms' advertising decisions. Advertising may also respond to a discouragement effect if one firm has significantly more market share and brand capital. Even a more productive follower will choose against advertising as advertisement will not drive significant market share. In this world, the less productive leader has incentives to advertise to limit the expansion of the more productive follower

These results paint a complex picture of the role of advertising in brand development and market share. On the one hand, advertising can indeed sort consumers to the most productive firm. On the other hand, advertising can serve to solidify the incumbent advantage for *less* productive incumbents than their competitors. The role of advertising depends on history, persistence, and the elasticity of substitution of consumers.

Overall, advertising in our model also exhibits similar features to the role of advertising discussed in the literature (Cavenaile et al., 2022). There is a role of *net* advertising which sorts consumers across firms, and *gross* advertising which uses up resources in the economy to try to capture the attention of consumers. The business stealing effect (the relationship between gross and net) thus plays a role in the social planner's decision, who would generally choose no advertising for firms with lower productivity in each market.

The overall nature of advertising has very similar flavors to the role of reallocation. Net advertising will shift brand capital across firms, while reallocation will also shift it across firms. In both cases, the incentives of firms with large brand advantage to maintain their advantage looms large.

Creative destruction occurs when a current productivity laggard gets knocked off by an entrant who improves upon the existing incumbent's productivity. The new entrant advances productivity, but has minimal brand capital. The dynamics of their interaction are essential for understand how growth affects these market dynamics.

While higher creative destruction creates turnover, which is good for growth, it also creates mismatch. An intuitive way to understand this is to think about very dynamic markets: in these markets, consumers have little awareness of which firm is the more productive firm and simply rely on the last name they've heard.

With the estimated model, we are ready to explore the quantitative implications of our framework. We do this in two steps. First, we decompose the main forces driving the variation in growth and concentration and discuss welfare implications. Second, we explore various policy counterfactuals related to innovation, reallocation, and antitrust policies.

We start by discussing the sources of growth (innovation, maturity, reallocation) through the lens of our model. We then turn to the sources of market concentration. These two forces present an important tension in the economy, and our policy analysis will explore this tension. On the second point, we analyze standard policies (e.g., blocking acquisitions, acquisition taxes and subsidies, entry subsidies) through the lens of our quantitative framework and evaluate their joint effect on growth, concentration, and consumer welfare. We focus on the main characterization under the homogeneous group estimation.

5.4 Role of Creative Destruction

Our current framework does not endogenize innovation and directs primary attention the development of brand capital. However, brand capital interacts with creative destruction in essential ways which we discuss in this section.

Brand Capital Mismatch. The interaction between brand capital and productivity has novel elements that interact with the rate of creative destruction in the market.

The downstream innovation response to brand reallocation is a function of the interaction between brand maturity and reallocation. As a result, to a first order approximation, policymakers can ignore the innovation effects of antitrust policy *when transactions are of mature brands*, because the discounted value of transactions to entering firms is low. However, there is a rising tendency for brands to exhibit shorter life cycles and become transacted earlier in their life cycle. For transactions early in the brand life cycle, the dynamic effects of reallocation become more relevant, as the option value of selling for an entering fringe firm becomes more relevant.

We discuss these results quantitatively here. Recall ι measures the speed of brand maturity. We evaluate the policy of shutting down reallocation with three benchmarks in Table 7.

Table 7: Counterfactual — Maturity and Efficiency w/ Shutting Down Reallocation

	Baseline	Fast Maturity ($\iota \times 10$)	Slow Maturity ($\iota/10$)
Change in Leader's Market Share (p.p)	-11.11	-9.23	-17.29
Change in Growth rate (p.p) (%)	-0.321	-0.982	-0.141
Welfare (BGP, p.p)	-1.930	-21.75	-0.023
Welfare (Transition, p.p)	-1.332	-19.36	-0.001

Notes: Three counterfactual maturity scenarios. Source: Author calculations.

From column (1) to column (3), we consider how a different maturity rate of brands (with $\iota = 4\%$ as the estimated baseline) leads to different market concentration and welfare incidence. We add two extreme cases in columns (1) and (3), one where a brand grows at an average of 0.4% per year until peak, and another where a brand grows at an average of 40% per year until peak. We then compare changes in the innovation cost for entrants (κ_e) and changes in the search cost for brand reallocation (κ_s, as a stand-in for an ownership transaction tax). The results are striking, and suggest the maturity channel cannot be ignored in innovation and antitrust policies.

When markets mature quickly (e.g. average growth of 40% to peak), there is a large growth and welfare cost to shutting down reallocation (22% welfare cost). This is because the policy has a larger effect on both entry and reallocation. When maturity is slower (e.g., average growth of 0.4% to peak), shutting down trade decreases the leader's market share with a minimal impact on welfare (close to 0%). This occurs because the decline in reallocation has very little effect on entry, but significantly reduces concentration.

As for policy recommendations, both across industries and over time policymakers need to understand the life cycle profile and the age distribution of transactions. If older brands are much more likely to be sold, the focus on transactions will weigh the markup and efficiency effects. The framework in our model can still be used to link market shares, efficiency, and markups.

Yet, in markets where young brands are reallocated, policymakers should note the interaction between entry and reallocation. Entry responds positively to reallocation, in particular if reallocation rates are linked to young brands (as in Table 7). If policymakers focus only on the predictions on sales and prices, they may miss the dynamic effects and induce efficiency losses by simply looking at the problem in a static setting.

6 Conclusion

Brand capital is a central component of the modern economy and shapes the market structure of firms. We study brand capital through the lens of brand reallocation, which plays a major role in sales concentration, market leadership, and efficiency. We employ a novel dataset on the universe of brands to unpack the role of brand reallocation and brand dynamics in the macroeconomy. Empirically, we find that brand creation plays a much larger role for small firms than for large firms, while brand reallocation plays a major role in determining large firms' market shares. For both, the life cycle of the brands they hold is a crucial component of their market shares.

To understand the efficiency implications of brand capital, we introduce a model of firm dynamics with productivity and brand capital and productive and strategic incentives. In our quantified model, large firms tend to be more efficient than smaller firms but have more pricing power through amassing brand capital. This efficiency-markup tradeoff leads to a natural tension between economic growth and market concentration, which we study with the estimated model. We estimate the model using our detailed data to study a set of relevant policy counterfactuals: how does shutting down or taxing brand reallocation affect consumer welfare and efficiency? How does subsidizing brand entry affect these outcomes? How do these policies interact with the brand life cycle?

We find taxes and blocking reallocation to large firms tend to reduce concentration and growth, leading to lower welfare. Subsidizing brand entry is a policy that can target the same concentration level with a positive impact on growth. Further, there is significant heterogeneity across product groups. If policy is coarse, taxes and subsidies on reallocation may decrease economic efficiency. If policy can be applied by group, there may be gains from subsidizing reallocation in some groups and taxing reallocation in others. However, for the same level of concentration, brand entry subsidies persistently appear to induce more growth and be more welfare-enhancing.

Empirically, one avenue for further research is to understand the long-run evolution of the market for brands and long-run changes in ownership structure. These findings would touch on important topical economic questions in innovation, concentration, and the role of intangible assets in firm dynamics. Understanding the brand-firm interaction is essential to understanding the trends in market shares and market dynamics. Theoretically, as the importance of brand capital continues to rise, frameworks that address the connection between brands, concentration, and growth will be essential for academic and policy discussions. We expect to see brands playing an important role in linking firm dynamics to market shares and the aggregate economy. With brands and productivity jointly determining market share, this framework provides a new foundation for understanding the determinants of market power, concentration, and public policies.

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Appendix

Our Appendix is in five sections, mirroring the structure of the text. Appendix A discusses the data background and general points about large firms and brand acquisitions. Appendix B discusses the empirical analysis connections to the literature and robustness. Appendix C discusses the theoretical proofs and expands on the firm's dynamic problem. Appendix D discusses the estimation. Appendix E discusses the general robustness of the quantitative results. For an updated Online Appendix discussing technical details, please see `liangjiewu.com/files/tm_pw_apx_oct22.pdf`.

A Data Appendix

This section addresses the set of data sources relevant for the analysis and the data examples that motivate our investigation. Section A expands on the details of the merge across datasets.

A.1 Data Merge Details

As discussed previously, our main merge links USPTO Trademark data with RMS Nielsen Scanner data. We proceed by linking firms and products separately. Our merge matches over 80% of sales-weighted products. Some problems still emerge with short-names. We use "tokens" and fuzzy matches to deal with the names. Firms and products follow similar procedures and we discuss them in turn.

Firms. For matching firms, we first standardize on a large set of firm tags, eliminating common firm words, e.g. "CORP", "INC", "ESTABLISHMENT").[1] We then take the cleaned and standardized name and match according to a tokenized bigram matching procedure.

Brands. By focusing on brands, we direct our attention to long-running products held by firms. USPTO Trademark data provides the "tm_name" or the name associated with a registered trademark. RMS Nielsen follows a similar format, which has a "brand_name". We join the two by employing a token name matching. For brand names, there are no further removals of tokens beyond the firm-level analysis.[2] For brand age, we focus on the "prior" brand, as in the broader brand umbrella of the production. For transacted brands, we observe the level of the transaction and focus on this.

Transactions. As we discussed previously, we leverage evidence from transactions in both USPTO and RMS Nielsen Scanner data. Overall, we get 20% of brand transactions from USPTO Trademark data and 80% of transactions from Nielsen. While there are more transactions observed in trademark data, there are some within firm transactions we drop, as we generate a text similarity threshold above which we do not consider transactions.

[1] The full list is here ('AB', 'AG', 'BV', 'CENTER', 'CO', 'COMPANY', 'COMPANIES', 'CORP', 'CORPORATION', 'DIV', 'GMBH', 'GROUP', 'INC', 'INCORPORATED', 'KG', 'LC', 'LIMITED', 'LIMITEDPARTNERSHIP', 'LLC', 'LP', 'LTD', 'NV', 'PLC', 'SA', 'SARL', 'SNC', 'SPA', 'SRL', 'TRUST', 'USA', 'KABUSHIKI', 'KAISHA', 'AKTIENGESELLSCHAFT', 'AKTIEBOLAG', 'SE', 'CORPORATIN', 'GROUP', 'GRP', 'HLDGS', 'HOLDINGS', 'COMM', 'INDS', 'HLDG', 'TECH', 'GAISHA', 'AMERICA', 'AMERICAN', 'NORTH', 'OPERATIONS', 'OPERATION', 'DIVISION', 'COMPAGNIE','INTERNATIONAL', 'NORTH AMERICA', 'InBev').

[2] Standardizations include removing any relevant firm names as discussed in the firms section, but does not do any further standardizations and tracks the token grams within each brand name.

B Empirical Appendix

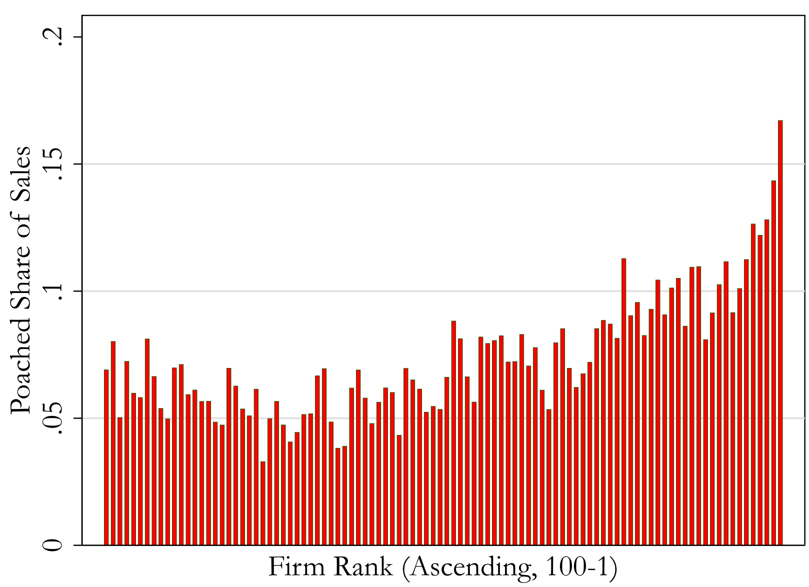
This section explores some additional evidence on a couple core messages from the paper, focusing in particular on the firm, brand, and firm × brand analysis. We apply broader data from the USPTO to indicate the fact that large firms build large portfolios of brands and their acquired brands drive a larger share of their portfolio. We then discuss the product life cycle with reference to the literature and discuss the integration of the product life cycle with our firm-level analysis. In each case, we explore the robustness of our results to varying definitions.

We start by expanding on the main elements of firm analysis in Section B.1, returning to the study of the sources of concentration, and evaluate the robustness of the empirical results on firms. We expand on the product life cycle in Section B.2, focusing on the interaction of age and sales, and the evidence for the importance of product maturity and sales dispersion over time. We further discuss our connection to the literature on the product life cycle and then turn to the robustness of product-level results. We then explore the event studies and the interaction of reallocation flows across firms in Section B.3. Lastly, we discuss the types of reassignment in the trademark data in Section B.4, which is in part a plea for further research to investigate further the sources and implications of IP reallocation.

B.1 Firm-Level Analysis

Figure B1 shows this pattern with respect to sales in Nielsen Scanner Data. We plot the share of sales from bought brands against the percentile (running from 1-100) of the firm size in sales.

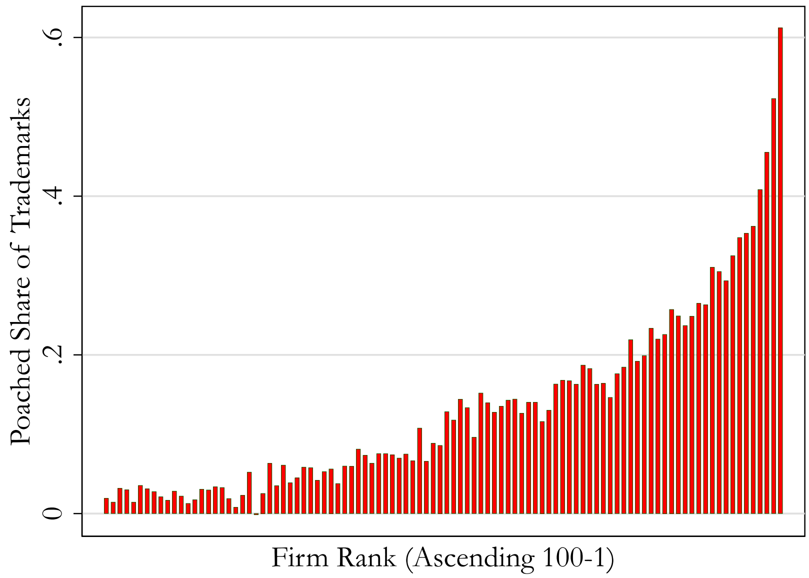
Figure B1: Contribution of Buying to Sales Share



Notes: Share of total sales from poached brands. Source: RMS Nielsen and USPTO Trademark

We find that the highest-selling firms have almost 4-times as much poached share of sales that a median firm, indicating that the pattern we find in the Trademark data on its own is consistent in the sales-share data. We observe this in both RMS Nielsen Scanner data and in USPTO Trademark data. Turning to USPTO data, we find the results are even more stark. Large firms tend to carry bought trademarks as a much larger share of their portfolio. This is noted in Kost et al. (2019), and can be seen in Figure B2.

Figure B2: Contribution of Buying to Trademark Stock



Notes: Market share reallocation measures across different firm size. Source: USPTO

We also run the following regression of three distinct margins of change, y_{it}, on the change of sales in each period $\Delta sales_{it}$:

$$y_{it} = \alpha + \beta \Delta sales_{it} + \epsilon_{it}. \tag{A1}$$

Equation (A1) focuses on three different margins for y_{it}, entry, maturity, and reallocation. We substitute y_{it} as the fitted value of sales on maturity ("Fitted Maturity") to understand how much of the observed variation from maturity is due to predictable life cycle growth. Table B1 evaluates the contribution of each force in the equation.

Table B1: Sources of Reallocation

	(1) **Entry**	(2) **Fitted Maturity**	(3) **Reallocation**	(4) **Unexplained**
Leader	0.033*	0.70*	0.13*	0.14
Fringe	0.091*	0.57*	0.021*	0.32

* $p < 0.001$

Note: Market share reallocation measures across different firm types, following Equation (A1). Source: RMS Nielsen.

We find similar patterns in Table B1. When we take fitted values from product-level age regressions, as discussed in the next section, the general pattern stays the same. We note that each force has a non-negligible contribution to the distribution of market shares, and our empirical model explains around 85% of the variation for large firms, and 70% of the variation for fringe firms.

B.2 Brand-Level Analysis

Method In order to understand the contribution of the firm and the brand to sales, we build on work that attempts to separate the firm and worker component of wages as in Bonhomme et al. (2019). This work proceeds in two steps.

First, the authors split firms into different types using a *k*-means clustering algorithm. Second, the authors identify off of movers across firm types the firm and worker effect (in our case, the firm and brand effect).If one ignores the brand, all variation would be assigned to the sales share of the firm. In the transaction, we can identify how the brand itself has persistence even when reallocated across firms.

In this section, we expand on the brand-level discussion in the main text, referring to brands and products interchangeably unless specifically indicated. Products are both a significant source of firm concentration (Hottman et al., 2016), yet highly dynamic (Argente et al., 2020b). The concentration implies a rich heterogeneity, but the dynamic nature implies that heterogeneity changes over time. The change in products can come from development of a product line or transactions of products from worse to better firms. Our goal in this section is to isolate the product element of the life cycle and show how even separate from the firms that hold them, products exhibit rich life cycles. This general point has been shown before (e.g. Argente et al., 2020b, 2021), but by integrating with USPTO Trademark data we are able to examine the longer brand life cycle and control for the transactions across firms.

Some products charge to dominance quickly, others rise gradually but maintain leadership, whereas others survive but remain in obscurity. Yet all brands must build customer capital to build market share. We direct our attention to brand *age* as a key ingredient to product market shares. We first focus on a snapshot of the distribution of sales by age, and then turn to an analysis of the life cycle to understand the more granular dynamics.

Products evolve over their life cycle. Gourio and Rudanko (2014) and Foster et al. (2016), among many others, have noted that customer capital is not built in a day. By looking at trademark data and Nielsen data, one can observe the importance of senior brands. Figure B3 takes data from 2016. We plot the brand percentile in terms of overall sales on the x-axis. On the y-axis, we plot the share of sales in this group that belongs to brands older than 10 years and brands younger than 10 years.[3]

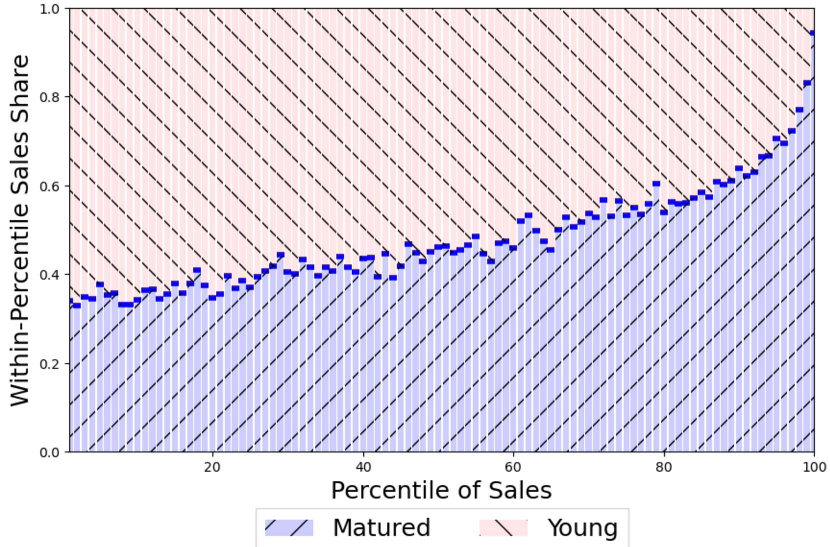
For brands created in 2006 and earlier, they maintain large sales share into the future. By 2016, those brands are still dominant in the top 1% of brands. Within the top 1% of brands, brands created before 2006 make up 92% of sales. Overall, old brands make up over 70% of sales, but only about 1/3rd of products. For the median brand in terms of sales, older brands make up less than half (38%) of total sales.

The dominance of mature brands could come from two forces. First, if few brands achieve such large sales, there may be a selection process. Young brands have less of a chance than old brands to have high customer capital, as the brands that survive to maturity must have a high quality draw. The composition only selects for the best. Second, brands could increase their sales over the life cycle such that only mature brands have significant sales share. We aim to understand this by linking a brand to its specific age. This is noted in the main text and Figure B12a. Yet, we are not the first to focus on this life cycle so we review the current literature benchmarks here.

Literature Benchmark: The Product Life Cycle. As discussed in the main text, our findings on the brand life cycle are significantly longer than the life cycle discussed in recent work (e.g. Argente et al.,

[3] We omit brands with less than $1000 in sales over an entire year, to have only brands that at least have a product line.

Figure B3: Brand Percentile and Maturity



Note: This figure shows the sales share within a percentile bin of products, split by those born before 2006 ("Matured") and after 2006 ("Young"). Source: RMS Nielsen Scanner Data.

2018). Here, we crosswalk our results to existing work on the product life cycle to benchmark where we diverge. Argente et al. (2018) focus on the life cycle of products applying Nielsen Scanner Data. This work is able to identify new products and brands and document their life cycle patterns. However, it is not able to link brands and products to their history, and is thus unable to speak to the longer time horizon of persistent brands. We perform similar life cycle regressions to the main text and compare them to a relevant current paper in the literature, in particular focusing on defining age in two different ways, to ensure the differences in the age profile does not simply come from applying a dataset with different age measures. Equation (A2) presents the regression:

$$\log y_{it} = \alpha + \sum_{a=0}^{4} \beta_a D_a + \gamma_b + \lambda_t + \epsilon_{it} \tag{A2}$$

Where the coefficients of interest are the coefficients on age (β_a) with controls for cohort and time effects (and an adjustment on cohort from Deaton, 1997). Table B2 engages in the same specification as Argente et al. (2018) in the UPC data (panels 1 and 2) and Trademark merged data (panels 3 and 4) respectively.

Note that while at the level of brands and trademarks there are significantly fewer observations, the same general pattern holds. This indicates how age is picking up something similar in our context, yet due to the broader horizon of historical data we are able to connect brands to their histories, indicating a significantly longer brand life cycle than found in Argente et al. (2018). We also show here similar general trends as in the main text when we evaluate the life cycle of products, controlling for brand-firm-group level.

Figure B5 evaluates the life cycle profile within a given product group code. We follow the regression in the main text, except in prices we weight by sales share. Equation (A3) illustrates the structure of the regression.

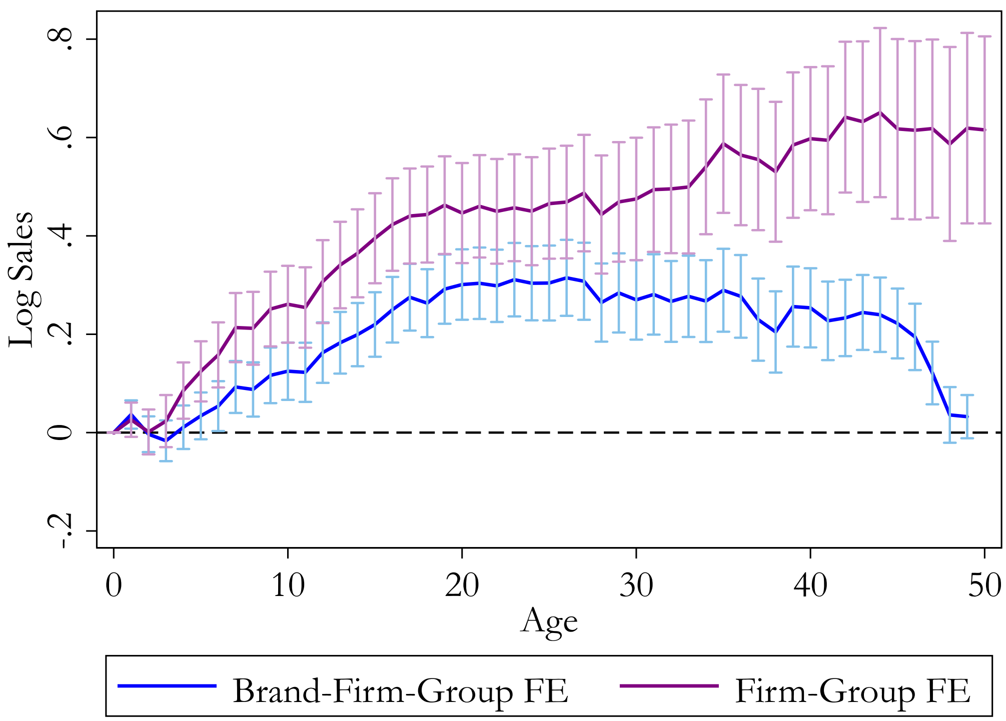
Table B2: Log Sales, by Nielsen and Trademark Age

	(1) Log Sales	(2) Log Sales	(3) Log Sales	(4) Log Sales
Age 1	0.939***	1.095***	0.917***	0.953***
	(0.00)	(0.00)	(0.00)	(0.00)
Age 2	0.857***	1.159***	1.019***	1.060***
	(0.00)	(0.00)	(0.00)	(0.00)
Age 3	0.632***	1.016***	0.834***	0.832***
	(0.00)	(0.00)	(0.00)	(0.00)
Age 4	0.169***	0.644***	0.412*	0.488***
	(0.00)	(0.00)	(0.00)	(0.00)
N	668993	89203	3402	4136
R^2	0.138	0.179	0.256	0.050
Variation	UPC	Brand-Group	TM Brand	TM Brand-Group

p-values in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: Balanced Panel Life Cycle Regressions of Log Sales on Age, utilizing different age sources and different variation. Source: USPTO Trademark and RMS Nielsen

Figure B4: Life Cycle Regressions

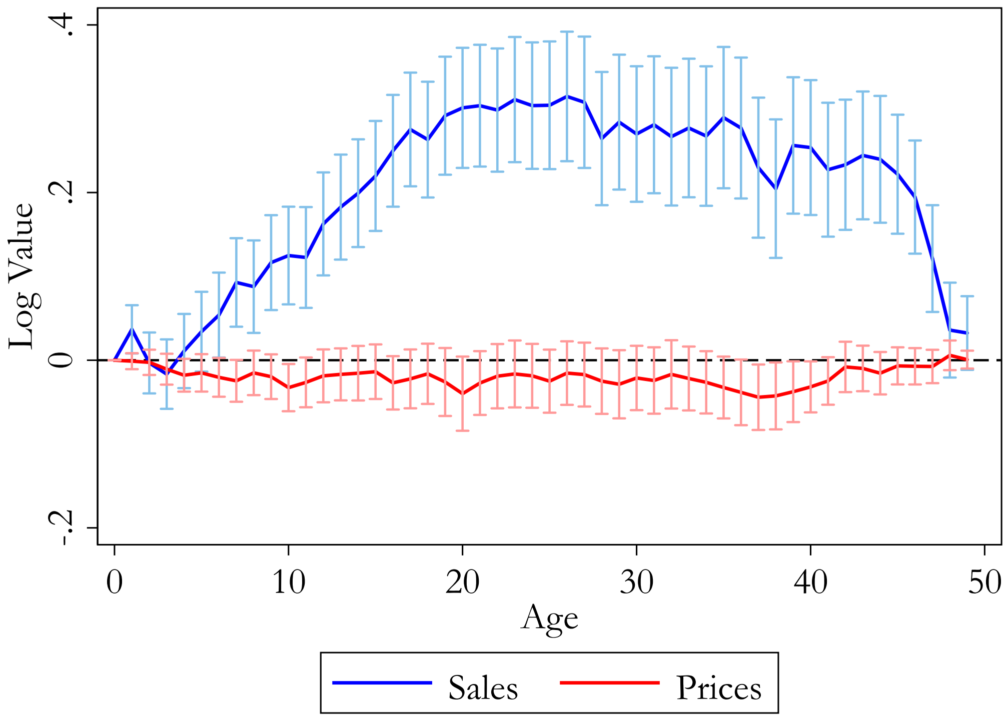


Note: Plots of log sales on age regression coefficients, controlling for brand-group and controlling only for firm-group. 95% confidence intervals plotted alongside coefficients. Source: RMS Nielsen and USPTO.

$$\log y_{ijkt} = \alpha + \sum_{a=1}^{50} \beta_a D_a + \gamma_b + \lambda_t + \theta_{ikj(i)} + \epsilon_{ijkt} \tag{A3}$$

The regression in Equation (A3) considers the sales and prices of brand i with firm j in group k at time t, $\log y_{ijkt}$ as a function of a constant (α), brand age indicators from 1 to 50, D_a, and fixed effects for cohort (γ_b) and time (λ_t).[4] The $\theta_{ikj(i)}$ indicates a brand-group or firm-group fixed-effect. Figure B5 plots the regressions by age coefficient β_a.

Figure B5: Life Cycle Regressions, Prices and Sales within Group



Note: Plots of log sales and prices on age regression coefficients, controlling for brand-group, as in Equation (A3). 95% confidence intervals plotted alongside coefficients. Source: RMS Nielsen and USPTO.

Figure B5 is consistent with the main facts from Section 3.2. We note that the inverted-U profile is still persistent within group, though with a slightly lower peak than in the brand's overall life cycle. We also note that the life cycle of prices shows on average somewhat minimal activity for the brand across age. This means that the strategic pricing firms engage in does not appear to be correlated with age, though as we have noted from events there are shifts in prices, consistent with previous evidence in the literature.

Definitions. In this section, we return to look at the qualitative similarities between the results in the main text and results depending on the definition of the firm and the main data used. For the main paper, we maintain the same dataset, focusing on brands with at least $1000 sales in a given year and brands that successfully merge to a trademark. Furthermore, given the nature of ownership we keep only the primary owner of a brand. This robustness section focuses on the empirical facts at the firm level addressing some changes to these definitions.

Product Definition. We focus on the product life cycle in our data, but aggregate across all brands in the main maturity specification to avoid brand $\times$ product group features. The life cycle peaks around

[4] Given the linear relationship between age, time, and cohort, we follow a method developed by Deaton (1997) to correct for this issue. The normalization orthogonalizes the cohort trends such that growth components move with age and time effects.

the same time in both specifications (see the peak age in Figure B12a and Figure B5). However, when we analyze brand $\times$ group, the life cycle peaks at a slightly younger level (0.35 versus 0.45). This should not change the qualitative implications of our results.

Transaction Definition. Transactions are defined at both the Nielsen and USPTO level. The reason we define transactions using both is as follows. We note that when we plot the results applying only USPTO transaction information we find as follows. Multiple serial numbers per brand.

B.3 Empirical Robustness: Firm $\times$ Brand Analysis

In the main paper, we focused on the responsiveness of sales and prices to both events and allocation to top firms. Here, we discuss different definitions of top firms and events to understand the general robustness of our results. We find qualitatively very similar results, which would not change the main messages of our analysis.

Prices and Sales at Top Firms. In this section, we explore varying the definition of a top firm to understand the differences in predicted sales. Table B3 focuses on the robustness of the higher log sales at larger firms. We see that larger firms tend to show higher sales of the same brand.

Table B3: Log Sales Conditional on Holding Firm, Trademark Age Fixed Effects

	(1) Log Sales	(2) Log Sales	(3) Log Sales	(4) Log Sales	(5) Log Sales	(6) Log Sales
Top 10 Overall	0.57***	0.55***				
	(0.000)	(0.000)				
Last Period Top 10			0.59***	0.69***		
			(0.000)	(0.000)		
Top 10 in 2006					0.47***	0.53***
					(0.000)	(0.000)
N	441300	3972	441300	3972	441300	3972
R^2	0.844	0.741	0.844	0.735	0.844	0.740
Weights	No	No	No	No	No	No
Restrictions	No	Only trans.	No	Only trans.	No	Only trans.

p-values in parentheses, clustered at brand-group level.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: This table documents two separate regressions on brands that are held by both market leaders (top 10 firm overall) and fringe firms, looking at the effect of leaders holding brands.

Table B4 focuses on the robustness of the higher log prices at larger firms, focusing only on the merged sample. We note that the results directionally hold, but exhibit a higher variace.

Gross Flows and Net Flows. One of the main aspects of our paper focuses on the reallocation of products across firms. We identify this reallocation by jointly using RMS Nielsen Scanner data and USPTO Trademark data.

Event Studies. Our event studies focus on transactions across firms in the data. For an observed transaction, both the buyer and the seller must exist in the data. We employ a balanced panel with seven periods. Given we use data from 2006–2018, we must restrict our event study analysis to brand transactions from

Table B4: Log Price Conditional on Holding Firm, TM age FE

	(1) Log Price	(2) Log Price	(3) Log Price	(4) Log Price	(5) Log Price	(6) Log Price	(7) Log Price	(8) Log Price
Top 10 Firm	0.33	0.26*			0.057	0.036		
	(0.177)	(0.062)			(0.170)	(0.403)		
Top 10 Firm in 2006			0.14	0.34*			-0.0091	0.029
			(0.350)	(0.088)			(0.869)	(0.516)
N	441300	3972	441300	3972	441300	3972	441300	3972
R^2	0.967	0.881	0.967	0.882	0.983	0.983	0.983	0.983
Weights	Total Wt.	Total Wt.	Total Wt.	Total Wt.	Period Wt.	Period Wt.	Period Wt.	Period Wt.
Restrictions	No	Only trans.	No	Only trans.	No	Only trans.	No	Only trans.

p-values in parentheses, clustered at brand-group level.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: This table documents two separate regressions on brands that are held by both market leaders (top 10 firm overall) and fringe firms, looking at the effect of leaders holding brands.

2009–2015. Due to some of the restrictions on our data, we focus on a broader definition of leading firms and flows from low-type to high-type firms. We explore the robustness of event studies depending on our characterization of an event study and definition of firm type.

To characterize flows that link fringe and leader buyers and sellers, we evaluate exchanges that move from smaller sellers to larger buyers, defined over the horizon of the sample. We make a couple of adjustments to the definition of a large firm to evaluate the robustness of our event study results.

Lastly, we compare more broadly the change in prices and sales upon the inflow of a brand to a large and small firm. We consider a large firm to be a top 10 firm within the product group code, and a small firm to be all other firms. We ask how prices and sales respond by doing the same analysis here. Limiting attention only to brands that move between firms, we also evaluate the price and sales differences depending on the holding firm in Table B4.

Figure B9 focuses on the different brand creation rates (entry as share of overall firm sales), and we note the much stronger entry rate of fringe firms than leaders.

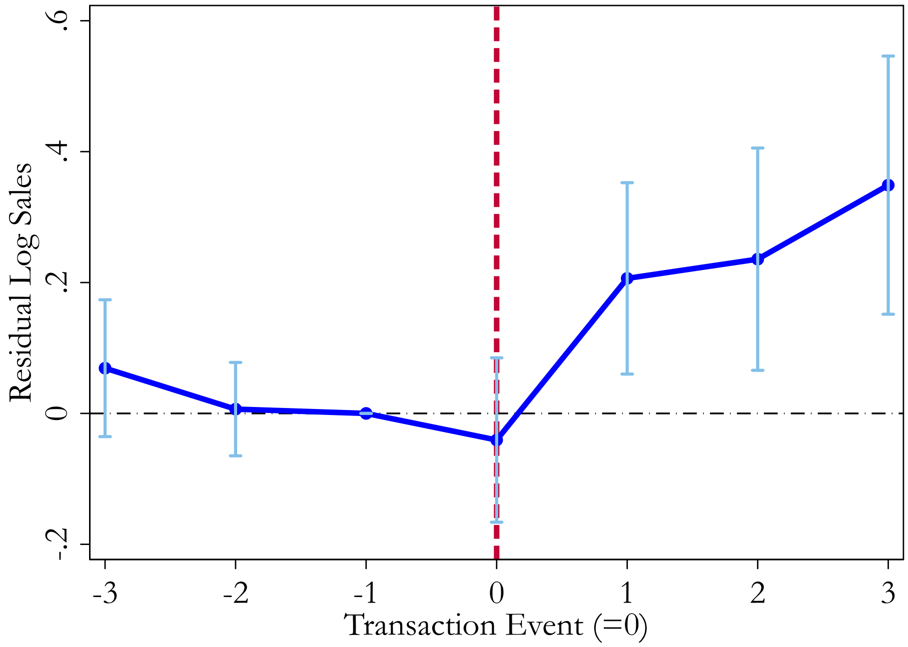
Further, we note that overall in the transfer of brand ownership there are more flows from small to large firms. This can be seen in firm press releases, as we observe many inflows and outflows of brand ownership for large firms, with inflows being more common. This can be seen in Figure B10, which collapsed the total brand flows (as share of sales) in both directions, with a histogram of net flows and line plots of gross flows.

B.4 USPTO Trademarks: Reassignment

The most reliable long-term data source for brand reallocation is USPTO Trademark data. Our focus in this paper is particularly on reallocation due to either pure reassignment (e.g. ownership transfer) or mergers & acquisitions. In this section, we discuss the general contours of the trademark data when it comes to reallocation of ownership. There is significant reallocation in the data, but some reallocation does not fall under the specific "merger" or "reassignment", but instead is linked to name changing, collateral, and other corrections and adjustments.

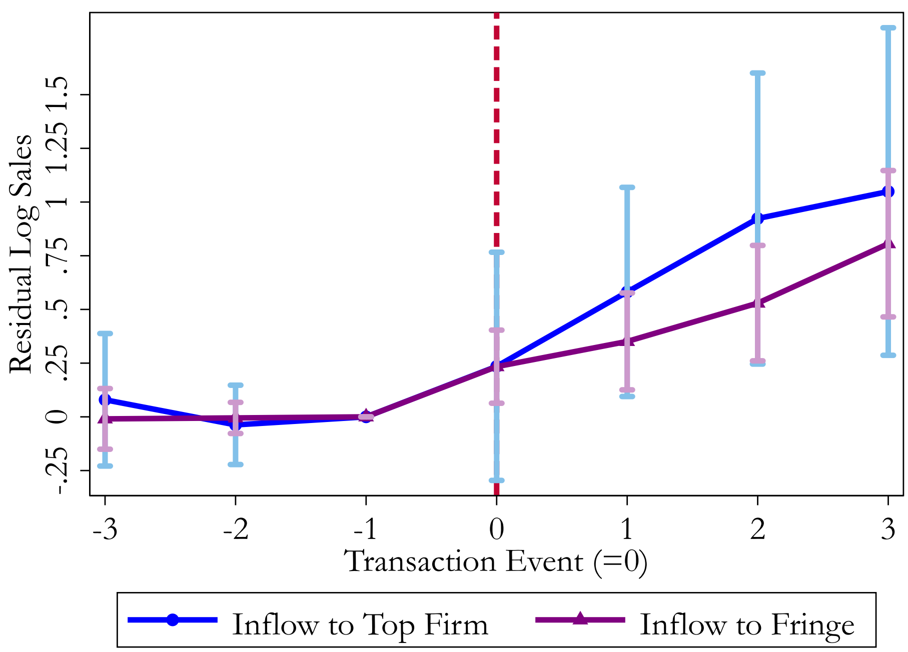
Table B5 splits the different transactions in the data into their different groupings. Most transactions in the data are available from 1970-2018. We order the transaction type by largest share of transactions. However, each transaction may contain a bundle of trademarks (e.g., transfer of ownership of "Odwalla"

Figure B6: Coarsened Exact Match and Brand Transaction Leader-to-Fringe, Log Sales



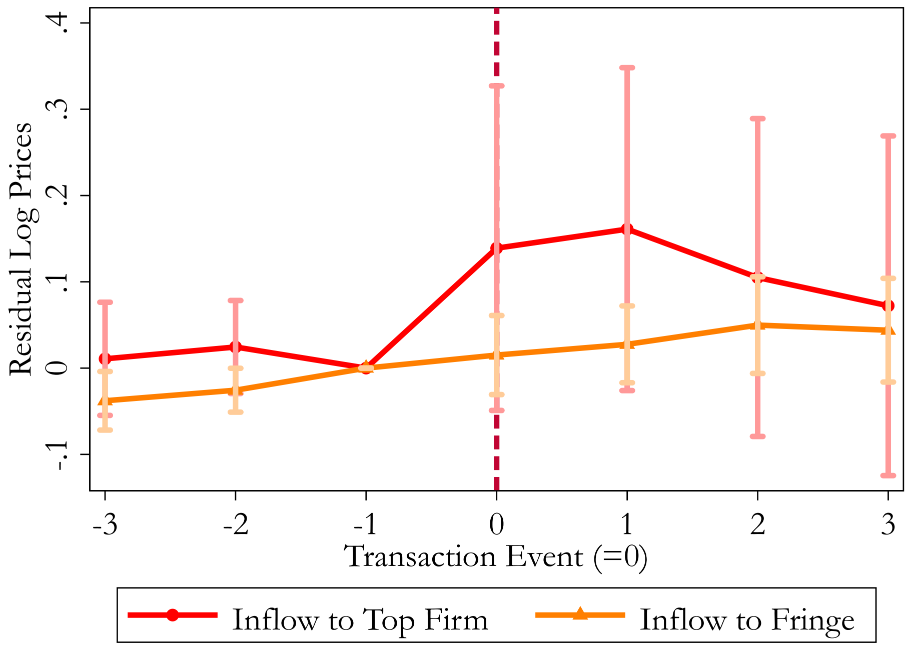
Notes: Coarsened exact match coefficients. Match is made on pre-trend sales change, brand age bin, and year.

Figure B7: Inflow to Top Firm and Fringe, Sales



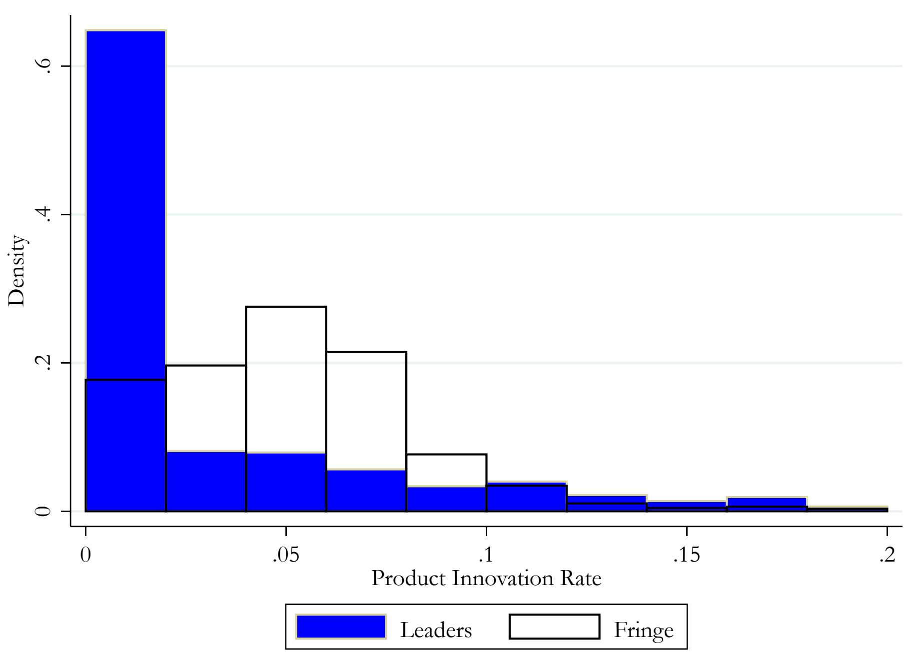
Notes: Coarsened exact match coefficients. Match is made on pre-trend sales change, brand age bin, and year.

Figure B8: Inflow to Top Firm and Fringe, Prices



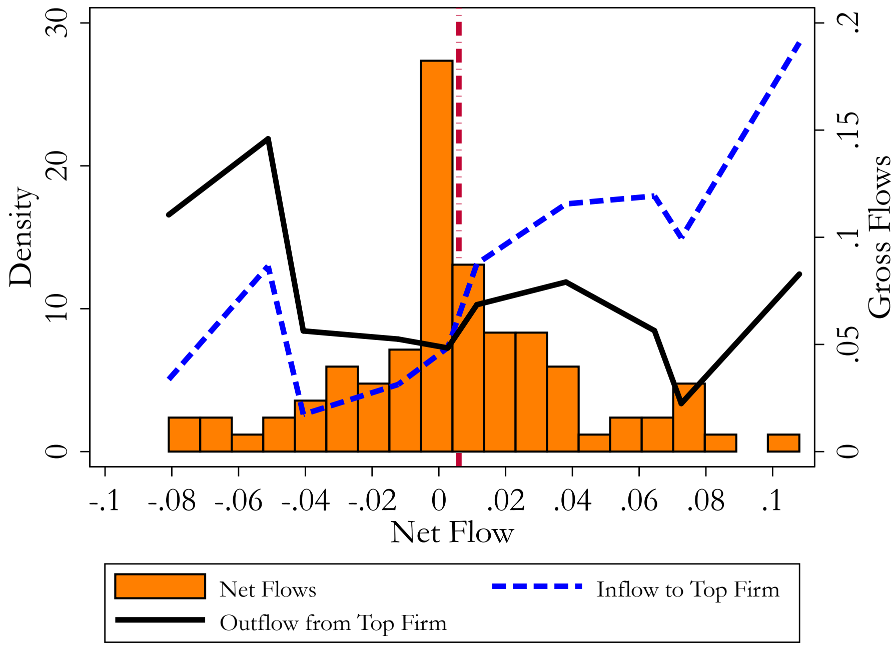
Notes: Coarsened exact match coefficients. Match is made on pre-trend sales change, brand age bin, and year.

Figure B9: Brand Creation by Type



Notes: This looks at the brand creation distribution by firm type, *Source:* RMS Nielsen and USPTO Trademark. Fringe and leader defined as in text.

Figure B10: Gross and Net Flows, Fringe and Leader



Notes: This looks at the market shares transferred across firms in market shares by year, averaged by product group code. *Source:* RMS Nielsen and USPTO Trademark. Fringe and leader (top 10 firm) defined as in text.

may be bundled with various sub-brands of the core brand Odwalla). For example, in the case of "Security Interest" (or collateral), note that on average a larger number of brands are involved in the pledged bundle.

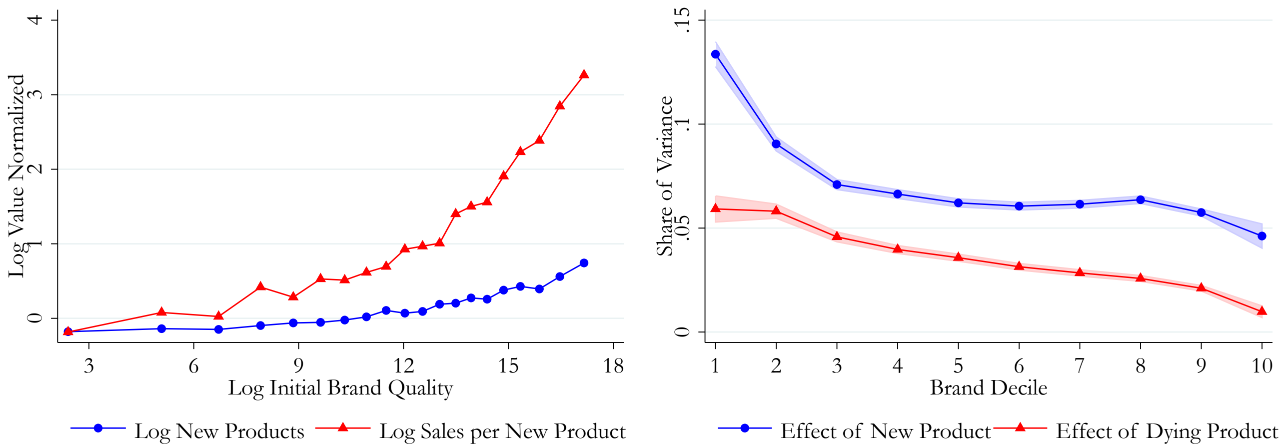
Table B5: Summary Statistics on Trademarks from USPTO

	Transaction Count	Trademark (TM) Count	TM/Transaction	Transaction Share	TM Share
Reassignment	478442	1.54M	3.21	**0.523**	0.345
Name Change	200767	795465	3.96	**0.219**	0.178
Security Interest	101280	1.10M	10.91	**0.111**	0.248
Merger	46610	287001	6.16	**0.051**	0.064
Correction	23500	119017	5.06	**0.026**	0.027
Other	64456	615334	9.55	**0.070**	0.138
Total	915055	4457996	4.87	1	1

Note: This table describes the category of each transaction in USPTO and orders them by their share of total transactions. Source: USPTO.

While our main focus in this paper has been mergers and reassignments, we note the richness of the data on multiple margins. Name changes are frequent, as firms may attempt to retool but maintain brand loyalty. Further, as noted previously, trademarks are often used as collateral. While Security Interest transactions are a small share of overall exchanges (around 10%), they make up almost 25% of all trademarks in exchanges. However, without transfer the firm may continue to operate these product lines. The benefit of focusing on mergers and reassignments is the reallocation of ownership and management across firms, but we hope to see further research on these margins.

Figure B11: Future Product Success and Brand Quality



(a) Product Entry and Brand Quality

(b) Sources of Brand-Level Variation

Notes: Figure B11a plots a binscatter of the relationship between new product entry and brand quality and log sales per new product against brand quality (log-log). Figure B11b plots the coefficients from a variance composition on the drivers of sales at the brand-level for entering products (blue) and dying products (red).

Fact 1: Brands Matter. Our first fact relates to the importance of brands for customer capital and market share. While there is a growing and significant literature on the role of customer capital in macroeconomics, the role of brands specifically as the locus of customer capital has received less focus. This section marshals evidence on the central aspect of brands in firm dynamics and market share.

B.4.1 Brands Matter: Mover Analysis

Table B6: Abowd et al. (1999) variance of firm log sales explained by brand

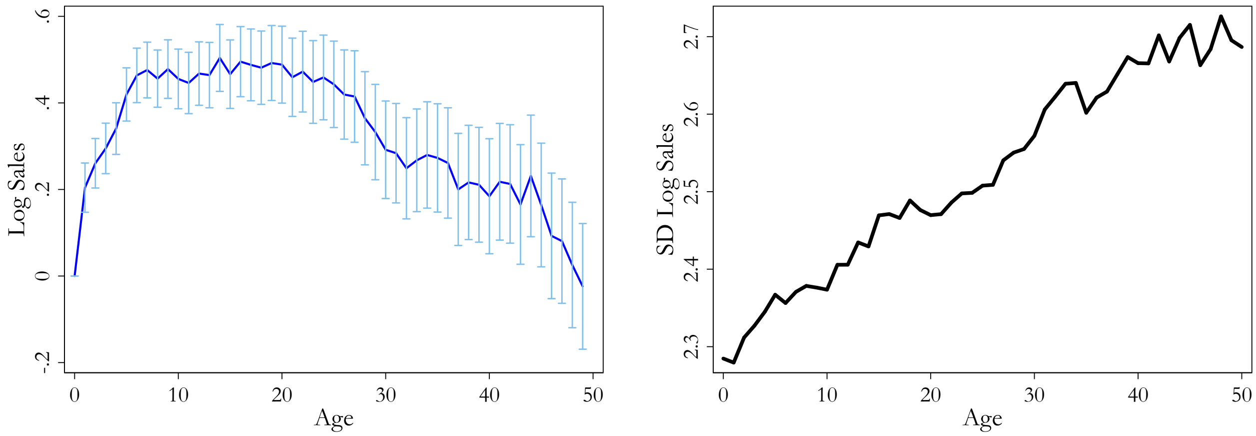
	10 bins	50 bins	100 bins
Brand	62%	61%	61%
Firm	18%	19%	19%
Brand × Firm	20%	20%	20%
Share Reallocated by Period	1.4% (overall reallocation 5.6%)[5]		
Top Firm Share	33.4%		
Median Firm Share	0.005%		

B.4.2 Birth/Death and Amplifiers

Fact 2: The Role of Reallocation. Brand reallocation across firms is a common feature of CPG markets and the market for trademarks more generally. 25% of market share reallocation comes directly from brand reallocation.

Fact 3: Event Studies. Here, we focus on the development of customers around the time of the event at the firm level.

Figure B12: Brand-Level Dynamics

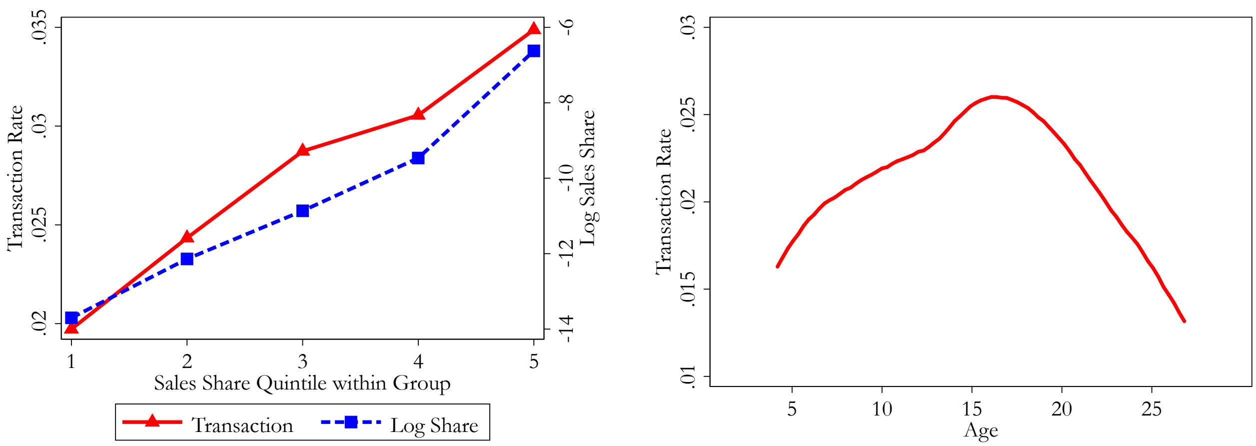


(a) Log Sales × Age Regression

(b) Log Sales Dispersion × Age

Notes: This figure plots a regression of log sales on age (panel a), and the standard deviation of log sales within age (panel b). 95% confidence interval standard errors clustered at the brand-group level. Source: USPTO Trademark Data and RMS Nielsen

Figure B13: Selective Reallocation



(a) Transaction Rate by Quintile

(b) Transaction Rate by Age

Notes: Panel (a): Transaction rate by sales share. Panel (b): Transaction rate by age. Source: USPTO Trademark and RMS Nielsen.

B.4.3 Entry and Exit at Customer Level

What does brand reallocation do for firms? In line with the evidence from Section 3.2, brands provide customer access for a firm, and firms can acquire brands to acquire the customer base connected to the brand. It has been noted that consumers have persistent preferences over brands (Bronnenberg et al., 2012), but this has not been studied with brand reallocation. To understand whether an acquisition of brands acquires customers, we track what happens to the customers at a firm and a brand when a brand is reallocated across firms. We find that a reallocation event is associated with an addition of new customers

Table B7: Customer-Level Entry

	(1) entry	(2) entry	(3) entry	(4) entry
reallocation_event	0.023*	0.023*	0.0076***	
	(1.75)	(1.78)	(4.48)	
l_reallocation_event				0.0085***
				(3.37)
_cons	0.32***	0.32***	0.32***	0.32***
	(26.07)	(26.50)	(476.52)	(438.45)
N	12050391	12050015	12049974	12049974

t statistics in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table B8: Customer-Level Exit

	(1) entry	(2) entry	(3) entry	(4) entry	(5) exit	(6) exit	(7) exit	(8) exit
reallocation_event	0.023*	0.023*	0.0076***		0.029**	0.027**	-0.0089***	
	(1.75)	(1.78)	(4.48)		(2.45)	(2.31)	(-4.82)	
l_reallocation_event				0.0085***				-0.0047***
				(3.37)				(-4.01)
_cons	0.32***	0.32***	0.32***	0.32***	0.39***	0.39***	0.40***	0.40***
	(26.07)	(26.50)	(476.52)	(438.45)	(32.49)	(32.86)	(565.86)	(519.55)
N	12050391	12050015	12049974	12049974	11181462	11180993	11180959	11180959

t statistics in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table B9: Customer-Level Entry

	(1) entry_u	(2) entry_u	(3) entry_u	(4) entry_u
reallocation_event	0.025***	0.026***	0.016***	
	(4.47)	(4.64)	(5.25)	
l_reallocation_event				0.015***
				(4.62)
_cons	0.74***	0.74***	0.74***	0.74***
	(429.65)	(440.78)	(17825.15)	(17994.78)
N	122868370	122868364	122704601	122704601

t statistics in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

to the firm.

Table B10: Customer-Level Exit

	(1)	(2)	(3)	(4)
	exit_u	exit_u	exit_u	exit_u
reallocation_event	0.0067	0.0067	0.0043**	
	(1.07)	(1.08)	(2.50)	
l_reallocation_event				-0.0015
				(-1.08)
_cons	0.75***	0.75***	0.75***	0.75***
	(452.38)	(462.52)	(33667.16)	(41385.78)
N	128882857	128882856	128713685	128713685

t statistics in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Productive and Strategic Effects: Sales and Prices. We observe brand transactions in the data and ask how prices and sales respond.[6] To ensure a relevant comparison group, we link transacted brands to never transacted brands with similar age, sales trends, and product group codes to the focal brands in this setting.[7] Both transacted brands and placebo brands are active for 7 years (3 years before event, event period, 3 years after), ensuring a balanced panel.

After the event, both prices and sales move strongly, with sales moving more. With the increase in prices, the results in Figure 3 provide evidence that both mechanisms, strategic acquisition and productive acquisition, could be at play.

Heterogeneity. While the average sales and price effects indicate an interesting tension between productive and strategic interaction, it masks a significant heterogeneity across transactions. Part of the motivating model presented indicated that depending on features of the market structure, such as productivity and the market share gap, some transactions may be efficient while others may be strategic. This tension can be found in the data, as some transactions exhibit what look like purely a productivity gain (only sales go up, and prices stay flat or decline), while others exhibit a purely strategic effect (prices go up, with a negative effect on sales). For instance, 14% of transactions exhibit a price change above the average and a sales change below the average. 18% exhibit a sales change above the average and a price change below the average.

Customer Acquisition We summarize the empirical results before turning to the model. We started this section by merging the data on products, brands, trademarks and firms. This dataset enabled a breakdown of the sources of market concentration, which we found to mostly come from the brands firms hold. Brands enable amplification of new products, drive product creation, and better brands are less affected by product death. Brands are constantly reallocated across firms, and the role of customer acquisition appears to be first order. There is evidence for productive (sales) and strategic (price), and this evidence is bolstered by the dominant role of large firms in specific markets. Finally, the role of heterogeneity should not be understated and will be essential in our model. Depending on the market structure, some brand

[6]We follow the same measurement of log sales and log prices in both the observed regressions and the event studies.
[7]We engage in a CEM distance matching but also do Mahabonolous matching which coarsens the sales into 30 bins in the pre-period. Similar results are found in both cases.

transactions can be efficient (better firm) and or strategic (larger but worse firm). These results motivate a model that can incorporate these forces to develop counterfactuals. Our model will incorporate these findings to characterize the drivers of concentration and the dynamic effects of brand reallocation. We turn to the model next.

C Theoretical Appendix

This section expands on some model discussion in the main text. Section C.2 expands on the leader's pricing decision in the main text. Section C.3 expands on the leader's dynamic problem in the main text, while Section C.4 expands on the equations and proofs in the main text.

C.1 Proof of Lemma 1

We first want to show $\Omega(b)$ is symmetric with respect to $-a$: $\Omega(b) = \Omega(-2a - b)$. We start by summing the value functions for the leader and the follower:

$$(\rho + \gamma + \lambda)\Omega(b) = \omega(b) + \gamma v(b_0) + \lambda \max_{\Delta}\{\Omega(b + \Delta) - R(|\Delta|)\}.$$

We have shown in the static equilibrium that $\omega(b) = \omega(-2a - b)$. With the guess $\Omega(b) = \Omega(-2a - b)$, the optimal choice of reallocation is identical for b and $-2a - b$. This verifies our guess.

Next, we want to show that $\Omega(b)$ increases when $b > -a$, and vice versa. Due to the symmetric, it suffices to show the statement is true for $b > -a$. Dividing through the equation by $\rho + \gamma + \lambda$ we get:

$$\Omega(b) = \frac{\omega(b) + \gamma v(b_0)\omega(b) + \gamma v(b_0)}{\rho + \gamma + \lambda} + \frac{\lambda}{\rho + \gamma + \lambda} \max_{\Delta}\{\Omega(b + \Delta) - R(|\Delta|)\}.$$

This is a Bellam equation for $\Omega(b)$. Because $\frac{\lambda}{\rho+\gamma+\lambda} < 1$, this equation is a contraction mapping. Becuase $\omega(b)$ is increasing for $b > -a$, the contraction mapping theorem implies $\Omega(b)$ is increasing for this domain. From symmetry, this also implies $\Omega(b)$ is decreasing for $b < -a$.

The function form of the reallocation cost implies the step of reallocation is always Δ. We start by writing out the joint surplus in the sequential form:

$$\Omega(b) = \max_{\Delta \in \{\Delta, -\Delta\}} \int_0^\infty \left[\int_0^T e^{-(\rho+\lambda+\gamma)t}(\lambda + \gamma)\omega(b)dt + e^{-\lambda T}\Omega(b + \Delta) + e^{-\gamma T}v(b_0) \right] dT$$

where we denote T the time when either the first reallocation event or the first creative destruction event happen. Suppose we start from $b_1 > b_2 > -a$. Because the choice of reallocation is not constrained, any policy of reallocation under b_2 is also feasible when the firms start with b_1. Thus

$$\begin{aligned}
\Omega(b_1) &\geq \int_0^\infty \left[\int_0^T e^{-(\rho+\lambda+\gamma)t}(\lambda + \gamma)\omega(b)dt + e^{-\lambda T}\Omega(b_1 + \Delta_2) + e^{\gamma T}v(b_0) \right] dT \\
&> \int_0^\infty \left[\int_0^T e^{-(\rho+\lambda+\gamma)t}(\lambda + \gamma)\omega(b_2 e^{\chi t})dt + e^{-\lambda T}\Omega(b_1 e^{\chi t} + \Delta_2(T)) + e^{\gamma T}v(b_0) \right] dT \\
&= \Omega(b_2)
\end{aligned}$$

Suppose $-a > b_1 > b_2$:

$$\begin{aligned}\Omega(b_2) &\geq \int_0^\infty \left[\int_0^T e^{-(\rho+\lambda+\gamma)t}(\lambda+\gamma)\omega(b_1 e^{\chi t})dt + e^{-\lambda T}\Omega(b_2 e^{\chi t} + \Delta_1(T)) + e^{\gamma T}v(b_0)\right]dT \\ &> \int_0^\infty \left[\int_0^T e^{-(\rho+\lambda+\gamma)t}(\lambda+\gamma)\omega(b_2 e^{\chi t})dt + e^{-\lambda T}\Omega(b_1 e^{\chi t} + \Delta_2(T)) + e^{\gamma T}v(b_0)\right]dT \\ &= \Omega(b_2)\end{aligned}$$

We can thus write out the joint surplus function as

$$\rho\Omega(b) = \Pi(a+b) + \pi(a+b) + \lambda[\max\{\Omega(b+\Delta), \Omega(b-\Delta)\} - \Omega(b)] + \gamma(v(b_0) - \Omega(b)) + \chi\Omega'(b).$$

To understand the direction of reallocation, we investigate the monotonicity of $\Omega(b)$.

C.2 Leader's Static Problem

Leaders attempt to maximize profits at each instant t. Recall the leader chooses prices subject to the demand curve as follows,

$$\max_{p_i} \int_{i\in\mathcal{I}_t^L} (p_i - e^{-z/(\sigma-1)}\mathbf{w}_t)c_t(p_i,\psi_i)di$$

s.t.

$$c_t(p,\psi) = \psi \times p^{-\sigma} \times P_t^{\sigma-1} \times \mathbf{C}$$

We include here the definition of the price index,

$$P_t = \left(\int_0^{N_t} \psi_{it} p_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}. \tag{A4}$$

To simplify notation, we omit the time subscript. The first order condition w.r.t. price for each product i is:

$$0 = \quad c(p,\psi)\left[1 - \sigma(p_i - e^{-z/(\sigma-1)}\mathbf{w}_t)\frac{1}{p_i} + (\sigma-1)\int_{i\in\mathcal{I}_t^L}(p_j - e^{-z/(\sigma-1)}\mathbf{w}_t)\frac{\psi_j p_j^{-\sigma}}{\left(\int_0^N \psi_i p_i^{1-\sigma} di\right)}dj\right] \tag{A5}$$

Using the definition of markups as price over marginal cost, $\mu_i = \frac{p_i}{e^{-z/(\sigma-1)}\mathbf{w}}$, we divide through by $c(p,\psi)$, and can re-write the first-order condition as:

$$0 = 1 - \sigma(1 - 1/\mu_i) + (\sigma-1)\int_{i\in\mathcal{I}_t^L}(1-1/\mu_j)\frac{\psi_j p_j^{1-\sigma}}{\int_0^{N_t}\psi_i p_i^{1-\sigma} di}dj \tag{A6}$$

Guessing that $\mu = \mu_j = \mu_i$:

$$0 = 1 - \sigma(1-1/\mu) + (\sigma-1)(1-1/\mu)\int_{i\in\mathcal{I}_t^L}\frac{\psi_j p_j^{1-\sigma}}{\int_0^N \psi_i p_i^{1-\sigma} di}dj \tag{A7}$$

$$= 1 - \sigma(1-1/\mu) + (\sigma-1)(1-1/\mu)s, \tag{A8}$$

where the second equality comes from the definition of leader's market share. Inverting this equation we can write the markup of leader as a function of its market share:

$$\mu = \frac{\sigma(1-s)+s}{\sigma(1-s)+s-1}.$$

Because the fringes can be viewed as a "leader" with zero market share, its markup is given by:

$$\bar{\mu} = \frac{\sigma}{\sigma-1}.$$

Lastly in order to solve for the market share of the leader, we use its definition:

$$s = \frac{Q_L\mu^{1-\sigma}}{Q_L\mu^{1-\sigma}+Q_F\bar{\mu}^{1-\sigma}} = \frac{\phi\mu^{1-\sigma}}{\phi\mu^{1-\sigma}+\bar{\mu}^{1-\sigma}}$$

C.3 Leader's Dynamic Problem

The leader chooses an innovation intensity (η), vacancies (o) and terms of trade (τ) to maximize the dynamic returns as follows,

$$\max_{\eta_t, o(\mathbf{x}), \tau^{LF}(\mathbf{x})} \int_0^\infty e^{-\int_0^t \mathbf{r}(t')dt'} \left[\Pi(\phi_t) - D(\eta_t) - B_t + S_t\right] dt, \tag{A9}$$

s.t.

$$\phi_t = \frac{\int e^{z+\alpha+\beta+\gamma} n_t^L(\mathbf{x}) d\mathbf{x}}{\int e^{\beta+\gamma} n_t^F(\mathbf{x}) d\mathbf{x}},$$

$$B_t = \int \left[M(v_t(\mathbf{x}), u_t(\mathbf{x}))\tau^{FL}(\mathbf{x}) - o_t(\mathbf{x})\right] d\mathbf{x},$$

$$S_t = \int \lambda(\theta_t(\mathbf{x}))\tau^{LF}(\mathbf{x}) n_t^L(\mathbf{x}) d\mathbf{x},$$

To characterize the optimal solution, we start by setting up the full Lagrangian:

$$
\begin{aligned}
&\mathcal{L}\Big(\eta_t, \nu_t(\mathbf{x}), \tau_t^{LF}(\mathbf{x}), v_t(\mathbf{x}), q_t(\mathbf{x}), \zeta_t\Big) \\
&= \int_0^\infty e^{-\int_0^t \mathbf{r}(t')dt'} \left[\Pi(\phi_t) - D(\eta_t) - B_t + S_t\right] dt + \int_0^\infty \zeta_t \left(\phi_t - \frac{\int e^{z+\alpha+\beta+\gamma} n_t^L(\mathbf{x}) d\mathbf{x}}{\int e^{\beta+\gamma} n_t^F(\mathbf{x}) d\mathbf{x}}\right) dt \\
&+ \int_0^\infty e^{-\rho t} v_t(\mathbf{x}) [\dot{n}_t^L(\mathbf{x}) - \underbrace{\eta_t f(\beta)\mathbb{I}_{\gamma=0}}_{\text{Innovation}} + \underbrace{\iota(\beta_0 + \bar{\beta} - \beta)\frac{\partial n_t^L}{\partial \beta}(\mathbf{x})}_{\text{Maturity}} \\
&+ \qquad \underbrace{\lambda\left(\theta_t^{LF}(\mathbf{x})\right) n_t^L(\mathbf{x})}_{\text{L-t-F Reallocation}} + \underbrace{\int_{\Omega(\mathbf{x},\tilde{\mathbf{x}})>0} f_\gamma(\gamma)\lambda\left(\theta_t^{FL}(\tilde{\mathbf{x}})\right) n_t^F(\tilde{\mathbf{x}}) d\tilde{\mathbf{x}}}_{\text{F-t-L Reallocation}} - g_t n_t^L(\mathbf{x})] dt \\
&+ \int_0^\infty e^{-\rho t} y_t(\mathbf{x}) [\dot{n}_t^F(\mathbf{x}) - \underbrace{\eta_t^F f(\beta)\mathbb{I}_{\gamma=0}}_{\text{Innovation}} + \underbrace{\iota(\beta_0 + \bar{\beta} - \beta)\frac{\partial n_t^F}{\partial \beta}(\mathbf{Q})}_{\text{Maturity}} \\
&+ \qquad \underbrace{\lambda\left(\theta_t^{FL}(\mathbf{x})\right) n_t^F(\mathbf{x})}_{\text{L-t-F Reallocation}} + \underbrace{\int_{\Omega(\mathbf{x},\tilde{\mathbf{x}})<0} f_\gamma(\gamma)\lambda\left(\theta_t^{LF}(\tilde{\mathbf{x}})\right) n_t^L(\tilde{\mathbf{x}}) d\tilde{\mathbf{x}}}_{\text{F-t-L Reallocation}} - g_t n_t^F(\mathbf{x})] dt
\end{aligned}
$$

We rewrite the integral with $\dot{n}_t^L$ using integration by part:

$$
\int_0^\infty \int_{\mathbf{x}} e^{-\rho t} v_t(\mathbf{x}) \dot{n}_t^L(\mathbf{x}) d\mathbf{x} dt = \int_{\mathbf{x}} \Big(v_\infty(\mathbf{x}) n_\infty^L(\mathbf{x}) - v_0(\mathbf{x}) n_0^L(\mathbf{x}) + \rho e^{-\rho t} v_t(\mathbf{x}) n_t^L(\mathbf{x}) - e^{-\rho t} n_t^L(\mathbf{x}) \dot{v}_t(\mathbf{x})\Big) d\mathbf{x}
$$

Similarly

$$
\int_0^\infty \int_{\mathbf{x}} e^{-\rho t} y_t(\mathbf{x}) \dot{n}_t^F(\mathbf{x}) d\mathbf{x} dt = \int_{\mathbf{x}} \Big(y_\infty(\mathbf{x}) n_\infty^F(\mathbf{x}) - y_0(\mathbf{x}) n_0^F(\mathbf{x}) + \rho e^{-\rho t} y_t(\mathbf{x}) n_t^F(\mathbf{x}) - e^{-\rho t} n_t^F(\mathbf{x}) \dot{y}_t(\mathbf{x})\Big) d\mathbf{x}
$$

For the choices to be optimal, any perturbation to distribution $n_t^L(\mathbf{Q})$ must yields no change to the Lagrangian. This implies:

$$
\begin{aligned}
(\rho + g_t)\, v_t(\mathbf{x}) =& e^{z+\alpha+\beta}\zeta_t \underbrace{\frac{Q_t}{Q_t^F}}_{=1+\phi_t} + \iota(\bar{\beta} - \beta)\frac{\partial v_t}{\partial \beta}(\mathbf{x}) + \dot{v}_t(\mathbf{x}) && \text{(A10)} \\
&+ \max_{\theta,\tau} \lambda(\theta)\mathbb{E}_{\gamma'}\Big[u_t(\mathbf{x}') + y_t(\mathbf{x}') - v_t(\mathbf{x})\Big] - \theta\kappa_s \frac{\mathbf{w}_t}{\mathbf{C}_t} && \text{(A11)} \\
& && \text{(A12)}
\end{aligned}
$$

For the choices to be optimal, any perturbation to distribution $n_t^F(\mathbf{Q})$ must yields no change to the Lagrangian. This implies:

$$
(\rho + g_t)\, y_t(\mathbf{x}) = -\, e^{z+\beta}\zeta_t\phi_t(1+\phi_t) + \iota(\bar{\beta} - \beta)\frac{\partial y_t}{\partial \beta}(\mathbf{x}) + \dot{y}_t(\mathbf{x}) \tag{A13}
$$

The other choices follow its first order condition:

$[\eta_t]$

$$D'(\eta_t)\frac{\mathbf{w}_t}{\mathbf{C}_t} = \mathbb{E}_{\beta_0} v\Big((\beta_0, 0, 0)\Big) \tag{A14}$$

$[\phi_t]$

$$\zeta_t = \Pi'(\phi_t) \tag{A15}$$

Combining these equations we reach the results in main text.

C.4 Model Proofs and Discussion

Search Process Discussion. In this section, we characterize the partial equilibrium in the search and matching markets, given (ϕ_k, Z_k) and the gains from reallocation across firms. Specifically, let $u_k(\beta, \gamma)$ be the discounted value of a fringe firm with product quality β and match quality γ, let $v_k(\beta, \gamma)$ be the discounted value of an additional product to the leader, and let $x_k(\beta, \gamma)$ be the discounted loss of an additional product operated by the leader in the calculation of leaders.

When positive buying flows into fringe firms occur, the optimal buying decision of a fringe firm with (β, γ) is as follows:

$$\kappa^s \varphi_0 = \max_{\tau} \frac{\lambda(\theta)}{\theta} \mathbb{E}_{\Delta}\Big[u(\beta, \gamma_L + \Delta) - \tau\Big]^+, \tag{A16}$$

s.t.

$$\lambda(\theta)\mathbb{E}_{\gamma_L}\Big[u(\beta, \gamma_L) - \tau\Big]^+ = U^F(\beta, \gamma).$$

It is straightforward to show Equation (A16) is equivalent to the following problem in terms of solutions:

$$U^F(\beta, \gamma) = \max_{\theta} \lambda(\theta)\mathbb{E}_{\gamma_L'}\Big[u(\beta, \gamma + \Delta) - u(\beta, \gamma)\Big]^+ - \theta\kappa^s\varphi_0 \tag{A17}$$

Equation (A17) provides an intuitive interpretation of the reallocation process: due to directed search and the competition on the buyer side, the terms of trade aims to maximize the net benefit of reallocating products from fringe firms to other fringe firms, taking into consideration of the search friction and the cost of search. It is also worth noting that for each (β, γ), Equation (A17) can be independently solved without referring to the distribution of products across firms. This mechanism is the block recursivity highlighted in Menzio and Shi (2011).

Similarly, due to free entry of fringe buyers, the leader-to-fringe (LtF) flows can be characterized in the same way. For notational simplicity, we define the joint surplus of reallocating a product from fringe to leader as $\Omega(\beta, \gamma_L, \gamma_F)$. The equilibrium in the LtF market is characterized by $\{U^L(\beta, \gamma), \theta^{LF}(\beta, \gamma)\}$ that jointly solve the following problem:

$$U^L(\beta, \gamma) = \max_{\theta} \lambda(\theta)\mathbb{E}_{\Delta}\Big[-\Omega(\beta, \gamma, \gamma + \Delta)\Big]^+ - \theta\kappa^s\varphi_0. \tag{A18}$$

The reallocation flow from the fringe to leaders is more complicated because there is no longer free entry on both sides of the market. However, the leader as a buyer faces competitive pressure from fringe buyers. In an equilibrium where flows are observed, the leader must offer the same expected value of selling as the fringe buyers. Thus, the optimal buying decision of the leader is

$$\kappa^s \varphi_0 \leq \max_{\tau} \frac{\lambda(\theta)}{\theta} \mathbb{E}_{\gamma_L} \Omega(\beta, \gamma', \gamma_F) - \frac{1}{\theta} U^F(\beta, \gamma_F). \tag{A19}$$

C.5 Derivation of Killer Acquisition Threshold

The killer acquisition threshold occurs when the value of selling a brand to the leader is negative, regardless of the efficiency differential between leader and fringe firm. This same intuition delivers a situation where leaders want to buy brands regardless of how efficient they would be at deploying the brand.

To theoretically study this situation, we focus on the relationship the leader and fringe have to a fringe firm's brand. We first take the difference between $u_t(\mathbf{x})$ (the value of brand to fringe firm) and $y_t(\mathbf{x})$ (the value of a *fringe's* brand to leader):

$$\begin{aligned}(\rho + g_t)\left[-y_t(\mathbf{x}) - u_t(\mathbf{x})\right] = & \underbrace{e^{\beta+\gamma}\left(1+\phi_t\right)\left[\Pi'(\phi) - \pi(\phi_t)\right]}_{\text{Operating Profit}} + \underbrace{\iota(\bar{\beta} - \beta)\left[-\frac{\partial y_t}{\partial \beta}(\mathbf{x}) - \frac{\partial u_t}{\partial \beta}(\mathbf{x})\right]}_{\text{Maturity}} \\ & + \underbrace{\max_{\theta} \lambda(\theta) \mathbb{E}_{\gamma'}\left[\Omega_t(\mathbf{x}', \mathbf{x})\right]^+ - \theta \kappa_s^{FL} \frac{\mathbf{w}_t}{\mathbf{C}_t}}_{\text{Value of Selling}} + \left[-\dot{y}_t(\mathbf{x}) - \dot{u}_t(\mathbf{x})\right]\end{aligned} \tag{A20}$$

First we find a threshold of ϕ such that $\Pi'(\phi) > \pi(\phi)\frac{1}{\phi}$:

$$(1 - s(\phi))\frac{\Pi(\phi)}{\phi} > \frac{1 - s(\phi)}{\sigma\phi}$$

$\iff$

$$\frac{s(\phi)}{\sigma(1 - s(\phi)) + s(\phi)} > \frac{1}{\sigma}$$

$\iff$

$$s(\phi) > \frac{\sigma}{2\sigma - 1}$$

Whenever the market share of leader is above this threshold, it must be $-y_t(\mathbf{x}) - u_t(\mathbf{x}) > 0$. Thus the gains from trade $\Omega(\mathbf{x}', \mathbf{x})$ is positive for any combination $(\mathbf{x}', \mathbf{x})$. As a result, there is never gains from trade of reallocating a product from leaders to fringes.

D Estimation Appendix

In this section, we discuss in greater detail the estimation process, starting generally and then discussing the different ingredients central to our estimation. 1. We directly calibrate the substitution elasticity σ_k to the ones estimated in the literature; 2. Given ϕ_k, we jointly estimate $\{\kappa_s, \kappa_e, d_k\}$ that minimize the distance between the observed reallocation rate, ϕ_k, and leader's innovation rate, as well as fringe firms' innovation rate.

D.1 Solving Equilibrium Given Parameters

Given any set of parameters, we take the following steps to solve the equilibrium, working at the group and aggregate level:

G1. *(Group Loop - Value Function)* For a fixed $\left(\frac{\mathbf{w}}{\mathbf{C}}, \phi, g\right)$, we solve the balanced growth path value functions and decisions according to the Bellman equations discussed in the main text. This can be done using any PDE solvers. We used the finite difference method;
G2. *(Group Loop - Aggregation)* Given the decisions, we solve for the BGP distribution, scaling the fringes' entry rate such that the BGP quality gap is consistent with the imputed ϕ. With this distribution, we calculate the residual in the free-entry condition and the residual in growth decomposition.
G3. *(Group Loop - Equilibrium)* We repeat step 1 and 2 such that the residuals on free entry condition and growth decomposition are both close enough to zeros.
A1. We repeat G1 - G3 for all groups, given a guess $\frac{\mathbf{w}}{\mathbf{C}}$. Using the aggregation results, we solve for the aggregate search labor, innovation labor, and production labor.
A2. Repeat A1 until the guessed $\frac{\mathbf{w}}{\mathbf{C}}$ are close enough to the one implied by labor supply curve.

D.2 Estimating Parameters

There are four parameters to estimate for each product group: two innovation costs and two search costs. For each set of parameter values, we solve the equilibrium, and calculate the aggregate innovation rate and reallocation rate for leaders and fringes. We find the parameters by the method of moments by minimizing the absolute norm between the model predicted rates and rates from data.

Elasticities, Shares, and Markups. At the inner layer, we need to establish the value functions of each agent and do value function iteration to link the shares and elasticities with the optimization problem of the leader and the fringe entry and selling decisions.

We start by specifying the leader's perceived elasticity, as discussed in the model, and in Equation (A21),

$$\epsilon(s) = (\sigma(1-s) + s). \tag{A21}$$

This simultaneously delivers a markup, of a leader with share s and a standard markup $\bar{\mu}$ for the fringe firm in Equation (A22),

$$\mu(s) = \frac{\epsilon(s)}{\epsilon(s) - 1} \;\; ; \;\; \bar{\mu} = \frac{\sigma}{\sigma - 1}. \tag{A22}$$

We also can specify the share as a function of the leader quality advantage ϕ, as follows:

$$s(\phi) = \max(1 + \phi^{-1}(\sigma/(\sigma-1)/\mu(x))^{1-\sigma}), (0,1)) \tag{A23}$$

$$\Pi_{fringe}(\phi) = 1/\sigma(1+\phi)(1 - s(\phi)) \tag{A24}$$

As a result, we can link the leader concentration ϕ to market shares and the elasticities firms face. This will represent the inner layer of our model, which occurs inside each iteration.

Full Discussion of Estimation. For more granular details of estimation, please see `liangjiewu.com/files/tm_pw_apx_oct22.pdf`

E Quantitative/Policy Discussion

In our quantitative exercises, we focus on different policies that seem to send a general message on brand reallocation. First, due to significant leader appeal and sales movement after exchange, we expect brand reallocation to show efficiency gains. We find this in the model, and find that downstream innovation also responds positively. Second, due to the age profile, the reallocation has less of an effect on growth than in markets with a faster age profile. Third, subsidizing entry is a more effective means of pursuing a reduction in concentration, as it simultaneously solves the growth and concentration externalities.

We believe that these results are robust to various specifications. First, on shutting down or taxing the reallocation of brands, we observe the responsiveness of brands to leader appeal is consistently larger than 0.4, while the marginal cost of leaders appear to be similar with fringe firms (Hottman et al., 2016). Leaders do engage in strategic behavior, but policy that shuts down reallocation will lose out on these gains and the forward looking behavior of fringe firms. This is attenuated if the leader appeal advantage declines or the brand maturity slows.

Second, as we see in Table 7, varying the maturity of brands has significant effects on policies, as faster maturity links innovation and reallocation more tightly together. This comes from directed search, and is consistent regardless of the life cycle characteristics, as long as there is some time to maturity, which is consistent with our paper and other work in the literature (e.g., Bronnenberg et al., 2009).

Third, subsidizing entry is a good policy for both attenuating concentration and increasing growth. This should hold as long as fringe firms have an innovation advantage (relative to their size) to leaders. If policy subsidizes product entry, both fringe and leading firms response, but fringe firms are able to respond more strongly. Even with reallocation, the steady-state share of fringe firms holdings are higher because reallocation occurs later in life. As a result, we feel the main messages of policy are robust to different specifications, but we look forward to further empirical and quantitative work to further explore these mechanisms.

Proof. We start by writing the surplus in its integral form:

$$\Omega(b) = \max_{B(a,b)} \mathbb{E}_0 \int_0^\infty e^{-(\rho+\lambda)t} \left[\omega(a_t, b_t) + \lambda e^{-\lambda t} \left(\Omega(a_t, B(a_t, b_t)) - r|B(a_t, b_t) - b|\right)\right] dt,$$

where the expectation is taken at time 0, given a. First we consider two value b_1 and b_2. Denote the associated optimal reallocation policy as $B_1(a,b)$ and $B_2(a,b)$. Consider a linear combination $\alpha b_1 + (1-\alpha)b_2$ with $\lambda \in (0,1)$. Consider a policy function that follows $B_1(a,b)$ with probability α and $B_2(a,b)$ with probability $1-\alpha$. Because this policy function is feasible given $\alpha b_1 + (1-\alpha)b_2$. It must be

$$
\begin{aligned}
&\Omega(a, \alpha b_1 + (1-\alpha)b_2) \\
\geq &\mathbb{E}_0 \int_0^\infty e^{-(\rho+\lambda)t}[\omega(a_t, b_t) \\
+&\lambda e^{-\lambda t}\left(\alpha(\Omega(a_t, B_1(a_t, b_t)) - r|B_1(a_t, b_t) - b|) + (1-\alpha)(\Omega(a_t, B_2(a_t, b_t)) - r|B_2(a_t, b_t) - b|)\right)]dt \\
\geq &\mathbb{E}_0 \int_0^\infty e^{-(\rho+\lambda)t}[\alpha\omega(a_t, b_t^1) + \alpha\omega(a_t, b_t^2) \\
+&\lambda e^{-\lambda t}\left(\alpha(\Omega(a_t, B_1(a_t, b_t)) - r|B_1(a_t, b_t) - b|) + (1-\alpha)(\Omega(a_t, B_2(a_t, b_t)) - r|B_2(a_t, b_t) - b|)\right)]dt, \\
= &\alpha\Omega(a, b_1) + (1-\alpha)\Omega(a, b_2)
\end{aligned}
$$

where the first inequality comes from the fact the proposed plan is feasible and the second inequality comes from the convexity of $\omega(a, b)$ in b.

■